MODAL DECOMPOSITION OF MEASURED VORTEX INDUCED RESPONSE OF DRILLING RISERS

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ABSTRACT

The paper presents a general method for modal decomposition of time series of structural vibration response. The method is applied to measurements of vortex induced vibrations of deep water drilling risers in operation. The modal decomposition method is based on a subspace system identification algorithm that is driven by the measured riser and rig response data only. Thus no a priori FEM model of the riser system is required for determination of the mode shape matrix that is used in the modal decomposition of the measured response time series. A combined deterministic-stochastic model of the riser-rig dynamic system is identified from response measured at fixed positions along the riser and from measurements of the rig motions. The rig motions are considered as a deterministic input to the dynamic system. The linear riser dynamics caused by the rig motions can therefore be separated from the riser vibrations caused by other unmeasured (or in this context by definition: stochastic) excitation sources, such as e.g. vortex shedding.

It is well known that hydrodynamic damping is significant for deepwater risers. Such damping is not classical in the sense of Caughey. The dynamic system model must therefore allow for general damping properties. This implies complex eigenvectors of the associated damped eigenvalue problem, and complex modal coordinates of the decoupled system. It will be shown how the elements of the complex eigenvectors can be interpreted in terms of magnitudes and phase angles of the corresponding mode shapes. The complex modal coordinate time series are interpreted as modal amplitude and modal phase angle. The modal decomposition method is illustrated by application to a few sets of measured response time series from a deep-water drilling riser.

KEY WORDS: System Identification, Modal Analysis, VIV, Riser

INTRODUCTION

The present trend in offshore petroleum exploration is towards deeper waters. The exploration drilling are very often carried out from a dynamically positioned floating vessel and the production solutions are also often based on a floating vessel. Riser systems provide the connection between the drilling or production vessel and the wellhead. Thus, the riser system integrity is vital for successful drilling and production. A major source of uncertainty with respect to design and operation of deep-water risers is the occurrence of Vortex induced Vibrations. As a part of the Norwegian Deep-water Programme (NDP), a drilling riser vibration-monitoring programme has been executed. The main objective of the monitoring programme was to establish a database that could be used for calibration of available VIV design software. A secondary objective was to gather data that could be used to enhance the understanding of VIV in deep-water risers.

MODELLING RISER DYNAMICS FOR SYSTEM IDENTIFICATION - PROBLEM FORMULATION

Interpretation of measured structural response requires an adequate mathematical model of the system. It is well known that hydrodynamic forces heavily influence the dynamic behaviour of risers. Especially the damping and the inertia contributions caused by the surrounding water may be as important as the structural damping and mass of the riser structure itself. Deep-water risers can therefore be regarded as neither lightly damped nor classically damped structural systems. Thus, the physical modal properties of deep-water risers cannot be explained by the theory of classical normal modes in the sense of Caughey (1960). Then it appears logical to base the interpretation of the measured resonant response on a more general theory of structural vibrations, namely the so-called “damped mode approach”. This theory allows more general mass, damping and stiffness properties than normally applied. The existence of such theory has been known for long, see e.g. Foss (1956), Hurty and Rubinstein (1964), Meirovitch (1967) and (1996), Langen and Sigbjörnsson (1979).

Measuring the dynamic response of deep-water drilling risers in operation is very challenging. The instrumentation must be attached to the riser during the riser assembly phase. This is a critical phase in the operation of the riser. Thus, in order to minimise time delays the instrumentation must be robust and simple. As few extra components as possible should be introduced into the riser assembly. The interference from non-drilling crew should also be minimised. Robit Technology, a Corrocean subsidiary, provides a solution that has been applied to a number of risers in the past couple of years. The equipment consist of independent motion measurement units which provides motion sensors, AD conversion, a fixed amount of data storage and battery capacity built into one high-pressure resistant titanium cylinder. Each unit has a clock, which can be synchronised initially with the clock of other units. However, since the clocks are independent there will be different drift for each of the clocks over the operation period. This drift leads to imperfect time synchronisation between the sampling performed by the different measuring devices. Assuming linear drift in time, an approximate correction may be performed. However, some phase distortion will remain in the measured response time histories.

Thus both the physical system that yields the response and the actual measurements of the riser response requires a general linearised dynamic model to be applied. Response measurements should therefore in general be processed and interpreted as coming from a system that permits spatial phase variations.

THE SECOND ORDER MODEL

The dynamic response of a marine riser can generally be modelled by a second order differential equation of dimension $(n \times n)$ as follows:

(1) $$\mathbf{M}_s\ddot{\mathbf{q}}(t) + \zeta_s\dot{\mathbf{q}}(t) + \mathbf{K}_s\mathbf{q}(t) = \mathbf{f}(\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}, t)$$

where $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$ are vectors of generalised acceleration, velocity and displacement, respectively. $\mathbf{f}(\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}, t)$ is the forcing function. $\mathbf{M}_s$, ζ_s and $\mathbf{K}_s$ are the mass, damping and stiffness matrices of the riser structure.

Consider a structure partly submerged in a fluid (e.g. water) and exposed to loading caused by fluid motions (e.g. waves and/or current) or the structure moving in the fluid as is the case for a riser connected to a vessel that moves due to wave and wind action. Then the forcing function $\mathbf{f}(\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}, t)$ can be decomposed into a sum of elements being proportional to the acceleration, velocity and displacement respectively and a residual which contain all the other load components, also any nonlinear effects:

(2) $$\mathbf{M}_s\ddot{\mathbf{q}}(t) + \zeta_s\dot{\mathbf{q}}(t) + \mathbf{K}_s\mathbf{q}(t) = \mathbf{f}_{\ddot{\mathbf{q}}}(t) + \mathbf{f}_{\dot{\mathbf{q}}}(t) + \mathbf{f}_{\mathbf{q}}(t) + \mathbf{f}_r(t)$$

The load component $\mathbf{f}_r(t)$ contains known (i.e. measured and thereby deterministic) excitation, $\mathbf{f}_r^d(t)$, such as e.g. vessel motions, and unknown (stochastic) excitation, $\mathbf{f}_r^s(t)$, such as wave and current action.

The first three elements of the right hand side of (2) are transferred to the left-hand side of the equation and expressed in terms of the acceleration, velocity and displacement respectively

(3) $$(\mathbf{M}_s + \mathbf{M}_h(t))\ddot{\mathbf{q}}(t) + (\zeta_s + \zeta_h(t))\dot{\mathbf{q}}(t) + (\mathbf{K}_s + \mathbf{K}_h(t))\mathbf{q}(t) = \mathbf{f}_r^d(t) + \mathbf{f}_r^s(t)$$

$\mathbf{M}_h(t)$ and $\zeta_h(t)$ are the hydrodynamic mass (inertia), and damping coefficients, while $\mathbf{K}_h(t)$ is an added stiffness due to hydrostatic effects. The hydrodynamic parts of the mass and damping matrices and the hydrostatic part of the stiffness matrix are generally not time invariant. Therefore the general equation will contain time varying coefficient matrices. However, it is reasonable to assume that they may be regarded as approximately constant. This is at least reasonable in a time scale related to the time characteristics of the system, i.e. natural periods. When the system during Vortex Induced Vibrations locks on to a natural period, it may be modelled with constant coefficient matrices. If the situation is inspected closely it may be seen that hydrodynamic added mass and damping at any given point on the riser usually show considerable variability over each cycle and between cycles presumably in the same state. However, the nearly harmonic response that is usually seen justifies the use of averaged quantities. Presumably the variability in hydrodynamic added mass and damping that is seen over short sections and time spans averages out over longer riser lengths and times. This is also justified by the fact that the system must have almost constant or very slowly varying coefficient matrices during the VIV lock-in period in order to have distinct natural frequencies. Then we obtain the following second-order differential equation

(4) $$\mathbf{M}\ddot{\mathbf{q}}(t) + \zeta\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{B}_r^d\mathbf{u}(t) + \mathbf{f}_r^s(t)$$

$\mathbf{B}_r^d$ is an input influence matrix characterising the locations and type of deterministic inputs $\mathbf{u}(t)$.

The response of the dynamic system can be measured by e.g. accelerometers, inclinometers, rotation rate sensors, strain gages etc. A matrix *output* equation can thus be written as:

(5) $$\mathbf{y}(t) = \mathbf{C}_a\ddot{\mathbf{q}}(t) + \mathbf{C}_v\dot{\mathbf{q}}(t) + \mathbf{C}_q\mathbf{q}(t) + \mathbf{e}_m(t)$$

where the matrices $\mathbf{C}_a$, $\mathbf{C}_v$ and $\mathbf{C}_q$ are output influence matrices for acceleration, velocity and displacement respectively. $\mathbf{e}_m(t)$ is white measurement noise. The output influence matrices describe the relationship between the vectors $\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}$ and the measurement vector $\mathbf{y}$. Thus, a measured output may be a combination of e.g. acceleration and rotation. This is in fact the case for accelerations measured with linear accelerometers mounted perpendicular to the riser axis. For motions with a long period, the influence of the acceleration of gravity (the "$g \cdot \sin(\theta)$" component) may exceed the lateral acceleration in magnitude. This needs special attention during analysis of the measurements.

A STATE-SPACE MODEL

Identification of the system parameters $\mathbf{M}, \zeta, \mathbf{K}$, which in modal form are given by natural frequencies, modal damping ratios and mode shapes are not straightforward. The system identification methods applied in experimental modal analysis today are to a large extent based on a reformulation of the second order model (4) into a first order state-space description. See e.g. Juang (1994). State space formulations have been applied for the purpose of system identification of offshore structures, see e.g. Hansteen (1987), Hoen (1991), Prevosto et al. (1991). Procedures for transformation of the second order model to state- space form can be found in textbooks on structural dynamics or system identification theory, see e.g. Hurty and Rubinstein (1963), Juang (1994) or Meirovitch (1996).

It is always possible to represent a linear system in state space form

(6) $$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{w}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{v}(t)\end{aligned}$$

With reference to (4) and (5) the following definitions apply

$\mathbf{x} = \begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix}$ is the state vector

$\mathbf{A} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\zeta\end{bmatrix}$ is the state transition matrix

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{B}_r^d \end{bmatrix}$$ is the deterministic input matrix

$$\mathbf{C} = \left[\mathbf{C}_q - \mathbf{C}_a\mathbf{M}^{-1}\mathbf{K}, \quad \mathbf{C}_v - \mathbf{C}_a\mathbf{M}^{-1}\boldsymbol{\zeta}\right]$$ is the output matrix

$$\mathbf{D} = \mathbf{C}_a\mathbf{M}^{-1}\mathbf{B}_r^d$$ is the deterministic feed-through matrix

$$\mathbf{v}(t) = \mathbf{C}_a\mathbf{M}^{-1}\mathbf{B}_r^d\mathbf{w}(t) + \mathbf{e}_m(t)$$ is the state measurement noise

$$\mathbf{w}(t) = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{f}_r^s(t) \end{bmatrix}$$ is the state process noise

In case the state process noise $\mathbf{w}(t)$ is not white, a state-space model can model the noise to yield a residual noise process that is white for practical purposes. This will add noise states to the state vector and corresponding terms to the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$. See e.g. Hoen (1991) for details.

A frequently applied alternative formulation to (6) in discrete time is the *innovation form*, see e.g. Ljung (1987)

$$\begin{aligned} \mathbf{x}(t+1) &= \overline{\mathbf{A}}\mathbf{x}(t) + \overline{\mathbf{B}}\mathbf{u}(t) + \mathbf{K}_g\mathbf{e}(t) \\ \mathbf{y}(t) &= \overline{\mathbf{C}}\mathbf{x}(t) + \overline{\mathbf{D}}\mathbf{u}(t) + \mathbf{e}(t) \end{aligned} \qquad (7)$$

where $\mathbf{K}_g$ is the Kalman gain matrix and the innovation is defined as $\mathbf{e}(t) = \mathbf{y}(t) - \mathrm{E}\{\mathbf{y}(t) \,|\, \mathbf{y}(t-1)\}$ where $\mathrm{E}\{\circ\}$ is the expectation operator. The system matrices $\overline{\mathbf{A}}, \overline{\mathbf{B}}, \overline{\mathbf{C}}, \overline{\mathbf{D}}$ are the discrete time equivalents of the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ of (6). The innovation formulation is particularly useful for estimating the state vector time series, since it is known to yield optimal estimates of the state vector. See e.g. Maybeck (1979).

THE STATE SPACE MODAL FORM

The state space models (6) or (7) can be decoupled into a set of *2n* uncoupled equations applying the eigenvalue decomposition of the state transition matrix

$$\boldsymbol{\Psi}^T\mathbf{A}\boldsymbol{\Phi} = \boldsymbol{\Lambda} \qquad (8)$$

where

$\boldsymbol{\Lambda}$ is the diagonal matrix of eigenvalues of $\mathbf{A}$

$\boldsymbol{\Psi}, \boldsymbol{\Phi}$ is the left and right eigenvector matrices of $\mathbf{A}$, ($\boldsymbol{\Psi}^T = \boldsymbol{\Phi}^{-1}$)

Thus we obtain the following modal state space description by applying (8) to e.g. (6)

$$\begin{aligned} \dot{\boldsymbol{\eta}}(t) &= \boldsymbol{\Lambda}\boldsymbol{\eta}(t) + \boldsymbol{\Psi}^T\mathbf{B}\mathbf{u}(t) + \boldsymbol{\Psi}^T\mathbf{w}(t) \\ \mathbf{y}(t) &= \mathbf{C}\boldsymbol{\Phi}\boldsymbol{\eta}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{v}(t) \end{aligned} \qquad (9)$$

where

$$\boldsymbol{\eta}(t) = \boldsymbol{\Psi}^T\mathbf{x}(t) = \boldsymbol{\Phi}^{-1}\mathbf{x}(t) \qquad (10)$$

is the complex vector of *state modal coordinates*.

It is well known that the solutions to (6), (7) and (9) are composed of a homogeneous part associated with the initial conditions, and a steady state solution given by the future deterministic input and process noise. The solution to the homogeneous part is useful for interpretation of resonant vibrations such as e.g. lock-in Vortex Induced Vibrations.

The eigenvector matrix of the state space model can be partitioned as

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_q \\ \boldsymbol{\Phi}_q\boldsymbol{\Lambda} \end{bmatrix} \qquad (11)$$

where $\boldsymbol{\Phi}_q$ is the components of the eigenvectors corresponding to the generalised displacements.

INTERPRETATION OF STATE SPACE MODAL RESPONSE

The solution to the free vibration problem associated with (6) is known to be

$$\mathbf{x}(t) = \boldsymbol{\Phi}e^{\boldsymbol{\Lambda}(t-t_0)}\mathbf{a}(t_0) \qquad (12)$$

where $\mathbf{a}(t_0)$ is a vector of complex coefficients or initial modal weights. Setting $t = t_0$ and premultiplying (12) with $\boldsymbol{\Psi}^T$ on both sides yields

$$\mathbf{a}(t_0) = \boldsymbol{\Psi}^T\mathbf{x}(t_0) = \boldsymbol{\eta}(t_0) \qquad (13)$$

We see that the complex modal weights are nothing but the state modal coordinates. In matrix form the free vibration state response is given

$$\mathbf{x}(t) = \boldsymbol{\Phi}e^{\boldsymbol{\Lambda}(t-t_0)}\boldsymbol{\Psi}^T\mathbf{x}(t_0), \quad t \geq t_0 \qquad (14)$$

Assume for simplicity of notation that $\boldsymbol{\Lambda}$ contains only complex eigenvalues, which then will appear in pairs as (λ_j, λ_j^*), where the asterisk denote complex conjugate. The free vibration response can then be expressed as the following sum over *n* components

$$\begin{aligned} \mathbf{x}(t) &= \sum_{j=1}^{n} \boldsymbol{\phi}_j e^{\lambda_j(t-t_0)}\boldsymbol{\psi}_j^T\mathbf{x}(t_0) + \boldsymbol{\phi}_j^* e^{\lambda_j^*(t-t_0)}\boldsymbol{\psi}_j^{*T}\mathbf{x}(t_0) \\ &= \sum_{j=1}^{n} \boldsymbol{\phi}_j e^{\lambda_j(t-t_0)}\eta_j(t_0) + \boldsymbol{\phi}_j^* e^{\lambda_j^*(t-t_0)}\eta_j^*(t_0) \end{aligned} \qquad (15)$$

Consider now the polar form of the complex numbers in (15)

$$\begin{aligned} e^{\lambda_j(t-t_0)} &= e^{-\alpha_j(t-t_0)} \cdot e^{-i\omega_j(t-t_0)} \\ \phi_{kj} &= |\phi_{kj}|e^{i\beta_{kj}}, & \beta_{kj} &= \arg(\phi_{kj}) \\ \eta_j(t_0) &= |\eta_j(t_0)|e^{i\theta_j(t_0)}, & \theta_j(t_0) &= \arg(\eta_j(t_0)) \end{aligned} \qquad (16)$$

Substituting for (16) in (15) results in the following expression for element *k* of the free vibration state response vector

$$x_k(t) = \sum_{j=1}^{n} 2|\eta_j(t_0)||\phi_{kj}|e^{-\alpha_j(t-t_0)}\cos(\omega_j(t-t_0) + \beta_{kj} + \theta_j(t_0)) \qquad (17)$$

The quantities that appear in (17) interpret as follows:

ω_j the damped natural frequency of mode j

$\alpha_j = \omega_j\zeta_j$ the damping coefficient, with ζ_j the modal damping ratio of mode *j*

$|\phi_{kj}|$ the magnitude of component k of right state eigenvector j

β_{kj} the phase of component k of right state eigenvector j

$2|\eta_j(t_0)|$ the initial modal amplitude of state mode j corresponding to the eigenvalue pair (λ_j, λ_j^*) and the initial condition $\mathbf{x}(t_0)$

$\theta_j(t_0)$ the initial modal phase of state mode j corresponding to the eigenvalue pair (λ_j, λ_j^*) and the initial condition $\mathbf{x}(t_0)$

Thus, a generally damped structural system decouples into n real state modes, each with $2n$ components corresponding to generalised displacements and velocities. The modes are defined by means of the complex eigenvectors of the system containing magnitudes and phase angles. The appearance of spatially varying phase angles admits travelling wave behaviour of the mode shape as the oscillation proceeds through a cycle. This is a major and important difference from the spatially synchronous oscillation found for classically damped systems.

We also see that the modal decomposition of a measured response vector time series $\mathbf{y}(t)$ can be obtained from estimates of the state-space system matrices and the corresponding state vector time series.

CIRCULAR SYMMETRIC STRUCTURE MODES

Circular symmetric structural systems such as e.g. risers do not have distinct principal axes. Therefore it is the hydrodynamic properties such as added mass and hydrodynamic damping that will determine the geometrical orientation of the riser mode shapes. The displacements and motions of the riser are typically given as three Cartesian coordinates $x_1 x_2 x_3$ with e.g. x_3 directed along the riser longitudinal axis. The column eigenvector ϕ_j can thus be split into three parts, $\phi_{1,j}, \phi_{2,j}, \phi_{3,j}$, in each of the orthogonal Cartesian directions. At an arbitrary position k of the riser, consider the free vibration displacement response at time $t \geq t_0$ caused by an initial state applied at time $t = t_0$ for the x_1-direction

$$x_{1,kj}(t) = 2\left|\phi_{q1,kj}\right| e^{-\alpha_j t} \cos\left(\omega_j t + \beta_{1,kj} + \theta_j(t_0)\right) \left|\eta_j(t_0)\right| \quad (18)$$

The subscript 1 has been introduced on the local magnitudes and phase angles of the state mode shape indicating that the component is measured along the x_1-axis. Analogous relations may be obtained by exchanging subscript 1 with 2 or 3 in (18). The motions in the x_3 direction of a riser are often negligible compared to the motions in the x_1, x_2 directions. Thus the resulting free vibration modal displacement response for a riser is obtained as

$$\begin{aligned} y_{kj}(t) &= \sqrt{x_{1,kj}^2(t) + x_{2,kj}^2(t)} \\ &= 2\left|\eta_j(t_0)\right| e^{-\alpha_j t} \sqrt{\sum_{\kappa=1}^{2} \left|\phi_{q\kappa,kj}\right|^2 \cos^2\left(\omega_j t + \beta_{\kappa,kj} + \theta_j(t_0)\right)} \end{aligned} \quad (19)$$

We see that the free vibration modal displacement response at an arbitrary position on the riser is the product of the initial modal amplitude, an exponentially decaying function and an ellipse. The exponentially decaying function is defined by the modal damping. The initial mode shape magnitudes, the damped circular eigenfrequency, the initial modal phase and the mode shape phase components define the ellipse. A similar expression is obtained for rotational degrees of freedom. The damping properties of the system, i.e. the damping coupling between the degrees-of-freedom in x_1 and x_2 directions, determine the ratio between the major axis and the minor axis of the ellipse. Therefore, the ellipse will degenerate to a line for systems with no damping coupling between the degrees-of-freedom in x_1 and x_2 directions.

The phase angle between a modal motion at two different positions k and l along the riser longitudinal axes is given as

$$\varphi_{j,kl} = \omega_j\left(t_{j,k} - t_{j,l}\right) \quad (20)$$

where $t_{j,k}$ and $t_{j,l}$ are the points in time relative to a reference time $t_{j,0}$ where the modal response of mode j reach maximum amplitude at the positions k and l respectively. This phase angle depends on the damping coupling between the degrees-of-freedom at position k and l.

A MEASURE ON MODE CONTRIBUTION

The state modal coordinate time series, (10), represent measures on mode contribution to the response from sample to sample. However, for identification purpose it is preferable to have a single number that yield similar information as can be obtained from the statistics of the state modal coordinate time series. Modal norms seem to be good candidates for such measures. Gawronski (1998) gives a comprehensive treatment of modal norms for system identification, model reduction and sensor placement purposes for stable systems. It can be shown, however, that results similar to the ones given by Gawronski also exist for non-stable systems in finite time intervals. This is due to certain properties of the analytical modal representation of Gramians as shown by Hoen (1991). Here we will only reproduce the results important for the present application.

Modal norms are related to system norms. System norms serve as a measure of system "size". The modal norm applied in the following is the Hankel norm. The Hankel norm of a system is a measure of the effect of the past input on the future output of the system. That is the energy stored in, and subsequently retrieved from the system. Thus the modal Hankel norm is a measure of the energy stored in the vibration of a mode. The modal Hankel norm is defined as follows for the present application

$$\gamma_i \cong \left|\frac{e^{-2\zeta_i\omega_i t} - 1}{4\zeta_i\omega_i}\right| \left\|k_i\right\|_2 \left\|c_i\right\|_2 \quad (20)$$

where

$\|k_i\|_2$ the Euclidean norm of the i'th row of the Kalman gain matrix of the stochastic model

$\|c_i\|_2$ the Euclidean norm of the i'th column of the modal equivalent output matrix of the stochastic model

ζ_i the estimated damping of mode i of the model

ω_i the estimated circular frequency of mode i of the model

The modal equivalent output matrix is the linear combination of three modal output matrices related to generalised displacements, velocities and accelerations as follows

$$\mathbf{C}_m = \mathbf{C}_{mq} + \mathbf{C}_{mv} + \mathbf{C}_{ma} \quad (21)$$

where

$$\mathbf{C}_{mq} = \mathbf{C}_q \mathbf{\Phi}_q \mathbf{\Omega}^{-1}, \quad \mathbf{C}_{mv} = \mathbf{C}_v \mathbf{\Phi}_q, \quad \mathbf{C}_{ma} = \mathbf{C}_a \mathbf{\Phi}_q \mathbf{\Omega},$$

$\mathbf{\Omega}$ is the diagonal matrix of natural circular frequencies and $\mathbf{\Phi}_q$ is the part of the right state space eigenvector matrix that corresponds to the generalised displacements, see (11).

The Hankel norm of a system is also the largest Hankel singular value of the system. Therefore it is close relations between the singular values of a system and the Hankel modal norms of the system. This property is useful for determination of model order in system identification.

IDENTIFICATION OF COMBINED DETERMINISTIC STOCHASTIC SYSTEMS

During the last decades considerable effort has been put into construction of algorithms for estimation of the parameters in MIMO (multi-input multi-output) state-space systems. In particular the so-called subspace methods, also known as projection methods, have drawn considerable interest. Over the years several authors have presented methods; for the deterministic case, the stochastic case and recently also for the combined case with both deterministic and stochastic input. See e.g. Ho and Kalman (1963), Kung (1978), Hoen (1991), Prevosto et al (1991), Juang (1994), Van Overschee and De Moor (1996), Ljung and McKelvey (1996), Di Ruscio (1997).

The basic idea behind subspace methods is to first estimate the state vector time series $\mathbf{x}(t)$, and then by linear least squares procedures estimate the system matrices $\overline{\mathbf{A}}, \overline{\mathbf{B}}, \overline{\mathbf{C}}, \overline{\mathbf{D}}, \mathbf{K}_g$. An estimate of the state vector time series may be constructed directly from the response measurements or from the corresponding covariance functions by application of standard linear algebra decompositions such as QR and/or SVD. See e.g. Golub and Van Loan (1996) for details on QR and SVD decompositions. From these decompositions, it is also possible to obtain the system matrices directly without actually computing the state vector time series.

Some of the algorithms presented in the literature are designed for impulse response type data, e.g. the ERA algorithm of NASA, see e.g. Juang (1994). Other algorithms can only handle stochastic systems and other again works for deterministic systems. However, the latest developments have led to a unification of the approaches and the construction of algorithms that handle the combined deterministic-stochastic estimation problem and each of them as well. The trend is also towards algorithms that work directly on the data avoiding the sometimes numerically ill conditioned covariance estimation step. See e.g. Di Ruscio (1997). Ljung and McKelvey (1996) give a relatively easy accessible and instructive introduction to sub-space system identification methods.

The DSR (Deterministic Stochastic Realisation) algorithm of Di Ruscio has some features that are very attractive for application to measured structural response. An estimate of the state vector time series can be obtained directly from standard linear algebra decompositions (QR and SVD) of a data matrix constructed from the input and output vector time series. The Kalman gain matrix is also computed directly from these decompositions without solving any matrix equations like e.g. Ricatti or Lyapunov equations. The state vector time series may therefore also be estimated applying a standard Kalman filter approach as an alternative to the direct estimation. We shall not go into details on the algorithm. The interested reader should consult Di Ruscio (1997) for a complete treatment.

We have performed some initial tests on riser response data applying the DSR algorithm, the N4SID algorithm of Van Overschee and De Moor (1996), the CVA algorithm of Larimore (1990) and the CBHM method given by Hoen (1991). These tests showed that the DSR algorithm generally performed better for identification of model order and system parameters than the other three algorithms. This confirms the results of Di Ruscio (1997) performed on quite other types of dynamic data.

MODEL ORDER SELECTION AND EVALUATION

Identification of models from measured data requires some judgement by the user. The number of excited modes is unknown and therefore also the systems order. A model estimated from the data will typically contain states related to excited structural modes, states related to the coloured excitation process and states related to noise and nonlinearites. A state that is related to an excited structural mode should be almost insensitive to increase in model order as long as the model order are not too far from the order of the true system. Thus if the estimates of the modal parameters corresponding to a state, i.e. the natural frequency, modal damping ratio and mode shape, stabilise with respect to increasing model order it is reason to consider this state as belonging to a true system mode.

Then we need a measure on mode coherence from one model order to the next. The inner product between the estimate of a mode for two different model orders is such a coherence measure. After normalisation of the mode vectors to unit length the inner product between the estimate of a mode at one model order and an estimate of the same mode at another model order should be close to unity. Estimates of corresponding modes obtained for different model orders that yield coherence not close to unity are either poorly excited system modes or noise modes. Each vibration mode can thus be characterised by frequency, mode coherence and damping ratio.

A stabilisation diagram visualising frequency location, mode coherence and damping level can be constructed as follows. The abscissa axis represents frequency and the ordinate mode coherence, while a colour may represent damping level. A vertical bar of length given by the mode coherence is plotted at the frequency location given by the corresponding frequency estimate. The colour of the bar can be used to represent the level of the damping ratio estimate. Starting with the model of lowest order several such diagrams can be stacked on top of each other and a mode stabilisation diagram is obtained. However, this diagram will not give information on the energy related to each of the excited modes. Similar stabilisation diagrams that yield this information can be constructed. Instead of scaling the length of the bars according to the mode coherence, the bar length can be scaled according to the relative size of the modal norms obtained for each model order. The bar length of the mode with largest modal norm for a particular model order is set to unity and the other bars are scaled accordingly.

An identification session thus consists of the following steps:

1. Determine a suitable model order and locate the excited structural modes by inspection of frequency stability diagrams.
2. Estimate the mode shapes and the corresponding modal amplitude time series for the chosen model order.

APPLICATION TO MEASURED RESPONSE

Some of the features of the method will be illustrated. Some typical riser response time series that demonstrate the use of the method and the results that may be obtained have been chosen.

The measurements have been performed on a drilling riser operating at a water depth between 1000m and 1500m. The external diameter of the riser including buoyancy elements are approximately 1 m and the average mass including content around 900 kg/m. The top tension was approximately 4.2 MN and the bottom tension was estimated to approximately 2.6 MN. The riser was equipped with five independent sensor units. In addition one sensor unit was located on the rig. The sensor layout is illustrated in Figure 1.

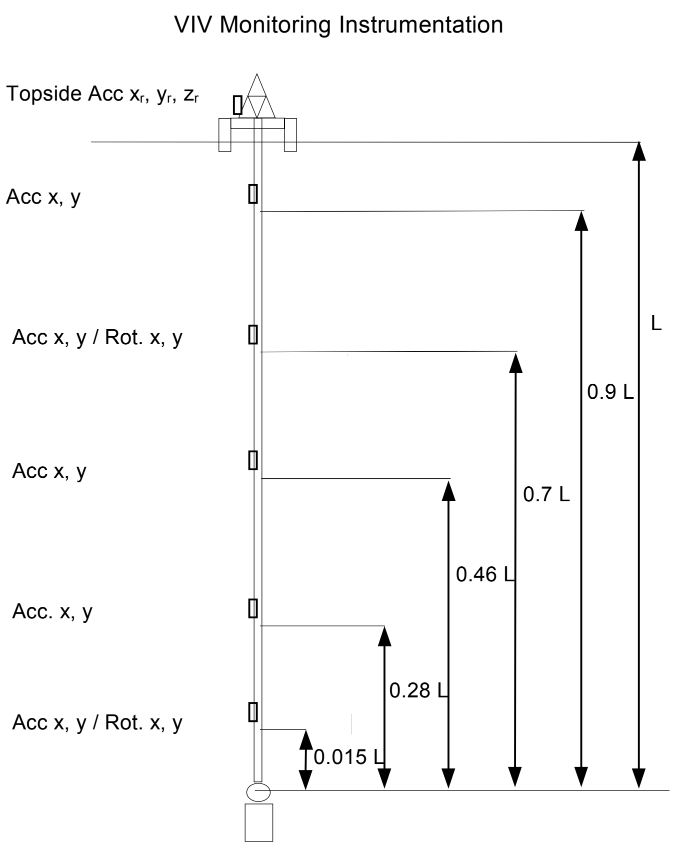

Figure 1 Sensor Positions

Only two of the sensor units were equipped with rotation rate meters. Thus it is only possible to separate the gravity-induced dynamic acceleration from the pure lateral dynamic acceleration at these two positions. At the lower sensor position, the motion is almost pure rotation. At this position, the accelerometer works as a dynamic inclinometer. The co-located rotation rate meter is therefore redundant.

Since the system identification works with the measured data, the coordinate system will be that of the accelerometers. This is fixed to the riser and moves with the riser as it vibrate. It is also some uncertainty related to the exact orientation of each sensor package relative to the others. The main reason is that the riser may twist slightly around its longitudinal axis. The results obtained from the system identification must therefore be interpreted with this in mind. Particularly this is important for interpretation of the mode shape estimates. Ideally one would like to compare the estimated mode shapes with the displacement modes of e.g. an FEM model of the riser. However, in the present case the comparison must be with mode shapes expressed in the sensor coordinates. I.e. each mode is expressed as a linear combination between the lateral acceleration mode and the rotation mode scaled with the acceleration of gravity as indicated in (22)

(22) $$\phi_m = \phi_a - g\,\phi_\theta = -\left(\omega^2 \phi_q + g\,\phi_\theta\right)$$

Another problem that arises with respect to comparison to riser FEM models is that the software applied for analysis of riser systems today only gives estimates of the modes of the undamped system. I.e. the influence from hydrodynamic damping is neglected.

Analysis of two cases with measurements will be presented. The accelerometer signals indicate one dominating mode with a low noise level for Case 1 while Case 2 apparently has two clear modes, but with a high-level coloured noise superimposed, see figure 2 and figure 7. The modal norm frequency stability diagram clearly identifies one mode for Case 1 and two modes for Case 2, see figure 3 and figure 9 respectively. The modal coherence frequency stability diagrams indicates two clear modes for both cases.

Table 1 Estimated Modal Parameters

Case	Period [s]	Freq. [Hz]	Damping ratio	Modal Norm	Modal amplitude [m]	
					Mean	COV
1	29.5	0.0339	0.0021	490 000	0.21	0.13
	22.8	0.0439	0.0054	88 000	0.05	0.56
2	37.8	0.0264	0.0055	280 000	0.41	0.39
	27.7	0.0361	0.0059	150 000	0.21	0.29

Table 1 shows the modal parameters estimated for the selected models of the two cases. We see that there is significant difference between the cases for the estimated natural period for both mode 1 and mode 2. The difference is largest for mode 1, which for Case 1 has a natural period that is approximately 78% of the natural period estimated for this mode in Case 2. This difference is far larger than what can be explained by difference in riser tension and mud weight between the two cases. Thus, there is reason to believe that the hydrodynamic added mass must be different for the two cases with a lower added mass for Case 1. We also see that the damping estimate for mode 1 in Case 1 is less than 50% of the damping level estimated for mode 1 in Case 2. From the modal norms we see that mode 1 is far more energetic for Case 1 than for Case 2. Mode 2 has almost insignificant energy for Case 1, but has an energy level of the same order of size as mode 1 for Case 2.

The estimated displacement modal amplitude time series for Case 1, see figure 5, show that the displacement response level of mode 1 is almost constant throughout the time series. This is also reflected in the Coefficient of Variation (COV) given in table 1. This indicates that the response may be of a steady state forced resonant type, like e.g. a lock-in VIV condition. Mode 2 starts out with a low response level which increases somewhat and then return back to the low level towards the end of the time series. We see that the COV for this mode is much larger than for mode 1.

The displacement modal amplitudes estimated for Case 2, see figure 11, show a much larger variability for mode 1 than found in Case 1. Mode 2 of Case 2 seems to be closer to a steady state response than mode 1, but not comparable to what was found for mode 1 in Case 1.

The mode shape estimates are shown in figure 6 and figure 7 for case 1 and in figure 12 and figure 13 for Case 2. We have compared the estimated mode shape magnitudes and phase angles to the corresponding mode shape magnitudes and phase angles computed by an undamped FEM model of the riser. The somewhat strange looking mode shape magnitude is obtained because we express the FEM mode shapes in the coordinate system of the accelerometers. Especially low frequency modes are significantly influenced by gravity as shown by eq. (22). Remember also that mode shape phase angles are relative.

For Case 1 it is relatively good agreement between the undamped FEM mode 1 and the estimates at the sensor positions except for the phase angle at the upper sensor level. For mode 2 the difference between the FEM mode and the estimated mode are larger both with respect to amplitude and phase angles.

For Case 2 it is good agreement between the magnitude of undamped FEM mode 1 and the estimate of mode 1 at the three lower sensor positions. At the two upper positions the deviation between the magnitude of the FEM mode and the estimated mode is significant. The phase angles differences are larger than for Case1. For mode 2 the agreement between the estimated mode and the FEM mode are better than for Case 1.

CONCLUSIONS

We have presented a modelling approach for the dynamic behaviour of deep-water risers that account for the influence of damping on the system dynamics. The modelling framework has been applied to obtain the modal decomposition of forced resonant vibrations of damped dynamic systems. The modal decomposition has been interpreted in terms of natural frequencies/periods, modal damping ratios, mode shapes with spatially varying phase angles and time series of complex state modal coordinates. The latter gives real valued time series of modal amplitudes and modal phase angles.

We have demonstrated the application of the modal decomposition method to full-scale measurements of riser acceleration response by using the DSR subspace system identification algorithm for estimation of realisations of the system matrices and the state vector time series.

The presented method seem to be a powerful tool for interpreting measured response from structural systems that respond dynamically to known and/or unknown excitation. In particular we expect that systematic application of the presented method to the available databases of measured riser VIV response may lead to a significantly enhanced understanding of the VIV response of risers.

We are presently investigating how to extend the method to obtain estimates of the unknown modal load from the decoupled system.

The method may also be adapted and implemented for online riser VIV monitoring purposes.

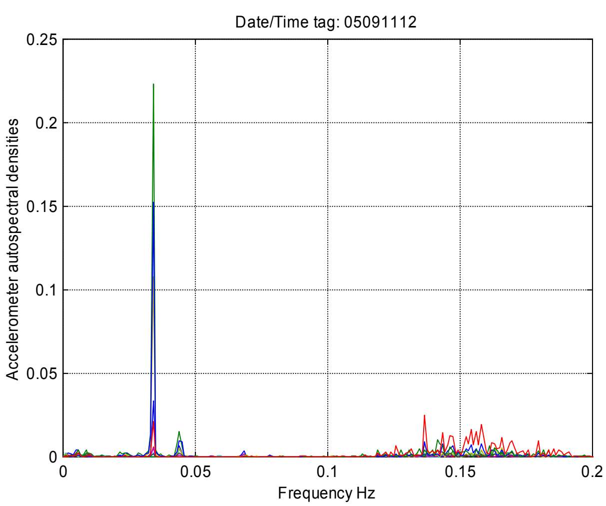

Figure 2 Raw FFT spectral estimates, Case 1

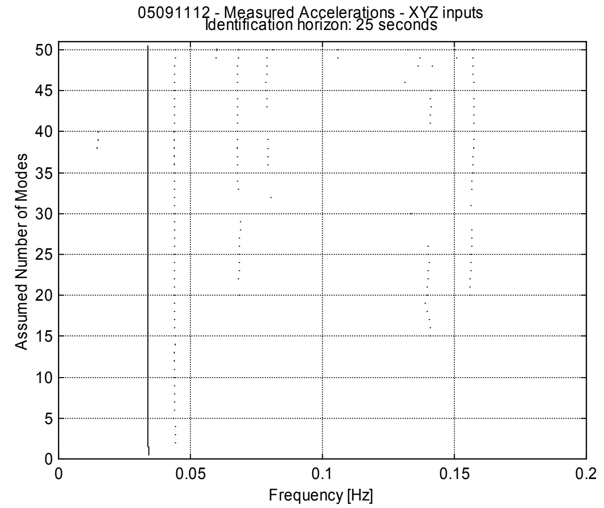

Figure 3 Modal norm frequency stability diagram, Case 1

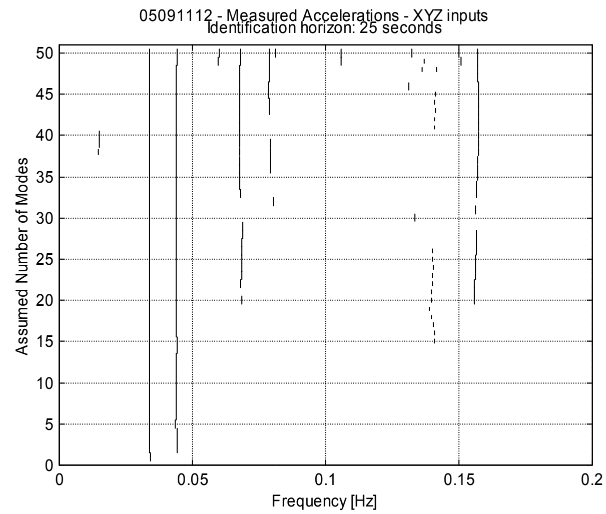

Figure 4 Mode coherence frequency stability diagram, Case 1

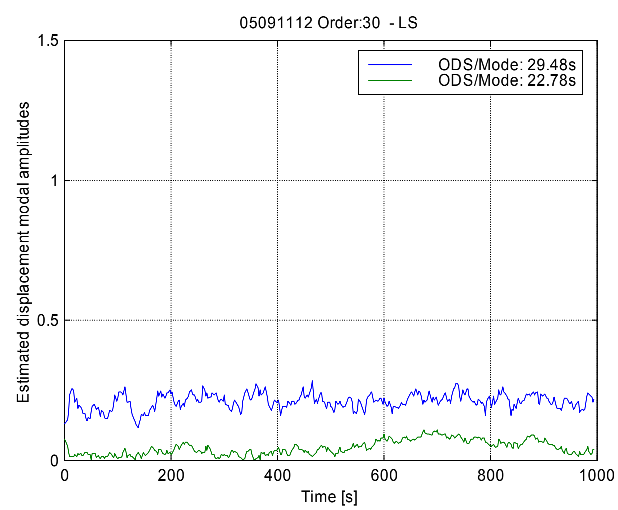

Figure 5 Estimated displacement modal amplitude [m], Case 1

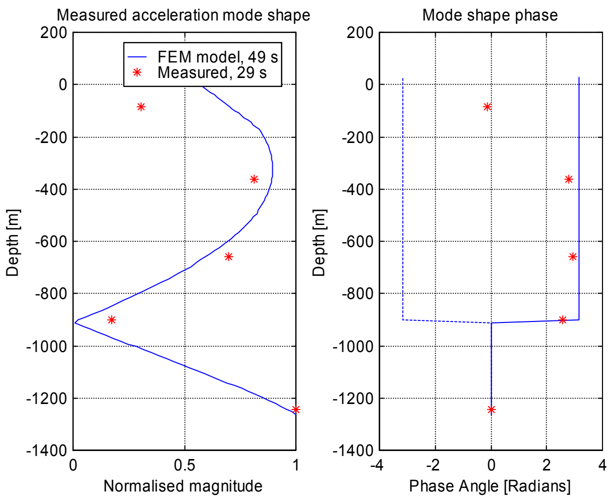

Figure 6 Comparison of FEM mode 1 with identified mode at 29 s

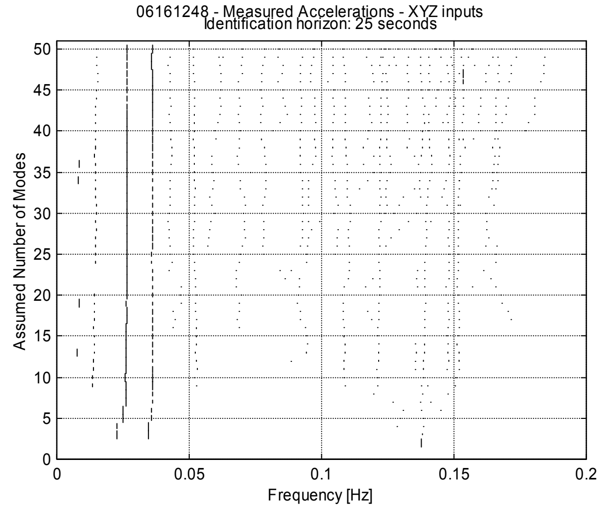

Figure 9 Modal norm frequency stability diagram, Case 2

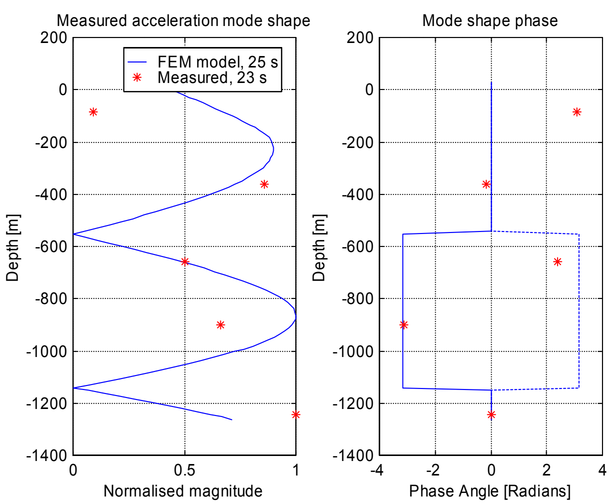

Figure 7 Comparison of FEM mode 2 with identified mode at 23 s

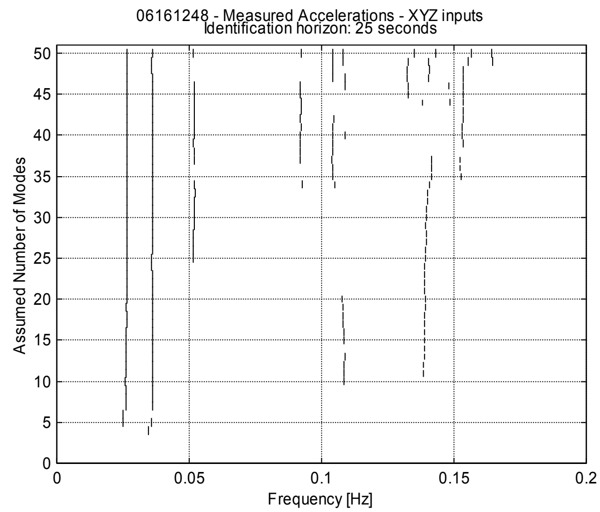

Figure 10 Mode coherence frequency stability diagram, Case 2

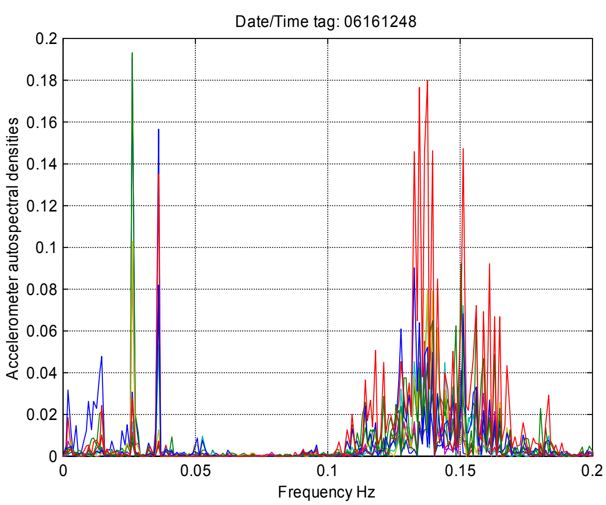

Figure 8 Raw FFT spectral estimates, Case 2

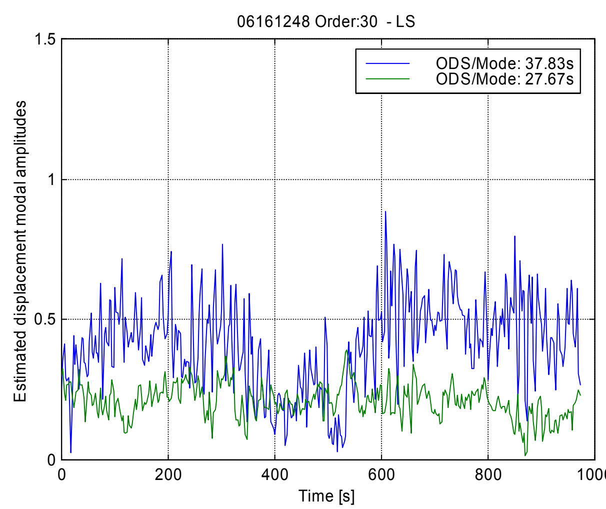

Figure 11 Estimated displacement modal amplitude, Case 2

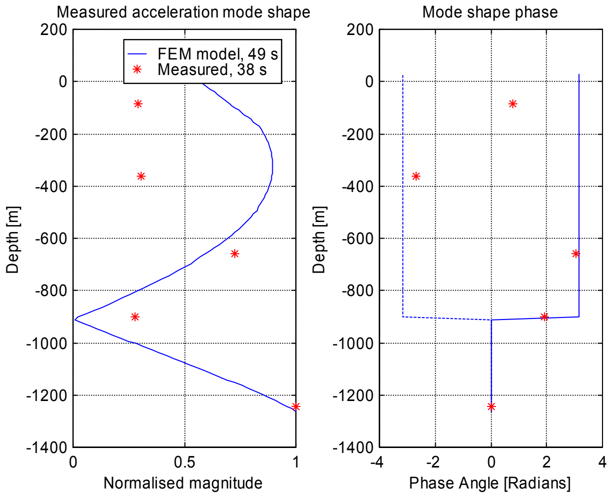

Figure 12 Comparison of FEM mode 1 with identified mode at 38 s

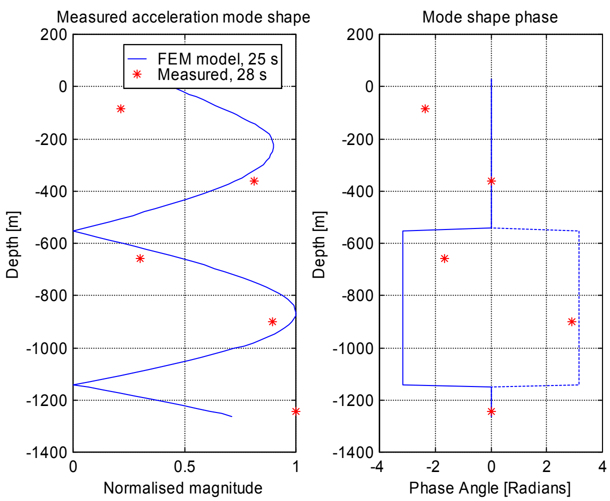

Figure 13 Comparison of FEM mode 2 with identified mode at 28 s

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