# Identification of Civil Engineering Structures using Vector ARMA Models

(*Identifikation af bygningskonstruktioner ved brug af vektor ARMA modeller*)

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# Contents

# Preface

In the mid 1980s, researchers at the Department of Building Technology and Structural Engineering at Aalborg University, Denmark, started using time domain models for system identification of civil engineering structures. In the past decade the results of this work have been reported in several papers and in three Ph.D. theses.

A common feature of the work has been the use of the so-called auto-regressive moving average (ARMA) models for time series modelling. The reason is the ability of these models to provide an accurate estimate of the modal parameters of a structural system on the basis of discretely sampled response.

However, the link between the ARMA model and the mathematical description of civil engineering structures has not been addressed to the same extend as mathematical description of dynamic systems in fields such as electrical engineering and econometrics. Therefore, the model has been applied as a grey-box model in the above references. The relation between the auto-regressive part of the model and the modal parameters has been well understood, whereas the understanding of the moving average part has been limited.

In order to obtain a deeper understanding of the ARMA models, and how they are related to the modelling of civil engineering structures, a Ph.D. project, with this as its primary objective, was granted as a part of the Danish Research Council frame programme "Dynamics of Structures". Another objective of the Ph.D. project was to implement the obtained knowledge as, especially designed, time domain software for system identification of civil engineering structures.

The present thesis *System Identification of Civil Engineering Structures using Vector ARMA Models* has been developed as a part of this Ph.D. project from September 1993 to May 1997 at the Department of Building Technology and Structural Engineering at Aalborg University.

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# Nomenclature

## General Rules

- Matrix and vector symbols are represented by bold type characters.
- Matrices are in general represented by capital letters.
- The transpose of a matrix is denoted by the superscript $^T$.
- The complex conjugate is denoted by the superscript $^*$.
- The complex conjugate transpose (Hermitan) of a matrix is denoted by the superscript $^H$.
- The inverse transpose of a matrix is denoted by the superscript $^{-T}$.
- The inverse hermitan of a matrix is denoted by the superscript $^{-H}$.
- Vectors are generally considered to be column vectors, and the scalar product is denoted by $\boldsymbol{x}^T\boldsymbol{y}$.
- Differentiation with respect to time is symbolized by superscript dots

$$\dot{\boldsymbol{y}}(t) = \frac{d\boldsymbol{y}(t)}{dt} \quad , \qquad \ddot{\boldsymbol{y}}(t) = \frac{d^2\boldsymbol{y}(t)}{dt^2}$$

- Continuous-time stochastic processes are denoted $\boldsymbol{y}(t)$.
- Discrete-time stochastic processes are denoted $\boldsymbol{y}(t_k)$.

## Scalars

| | |
|---|---|
| $\delta(\cdot)$ | Dirac / Kronecker delta function. |
| $f$ | Natural frequency. |
| $m$ | State space dimension. |
| $n$ | Order of a difference equation system. |
| $N$ | Number of data points of a single channel. |
| $na$ | Order of the Auto-Regressive matrix polynomial $\boldsymbol{A}(q)$. |
| $nb$ | Order of the Moving Average matrix polynomial $\boldsymbol{B}(q)$ / Number of columns of the input state space matrix $\boldsymbol{B}$. |
| $nc$ | Order of the Moving Average matrix polynomial $\boldsymbol{C}(q)$ / Number of rows of the output state space matrix $\boldsymbol{C}$. |
| $nk$ | Number of columns of the Kalman gain matrix $\boldsymbol{K}$. |
| $p$ | Number of output channels. |
| $T$ | Sampling interval. |

## Vectors and Matrices

| | |
|---|---|
| $\boldsymbol{A}$ | Transition matrix (Discrete-time state matrix). |
| $\boldsymbol{A}(q)$ | Autoregressive matrix polynomial. |
| $\boldsymbol{A}_i$ | $i$th auto-regressive coefficient matrix. |
| $\boldsymbol{B}$ | Stochastic input matrix. |
| $\boldsymbol{B}(q)$ | Moving average matrix polynomial (Noise-free case). |
| $\boldsymbol{B}_i$ | $i$th moving average coefficient matrix. |
| $\boldsymbol{C}$ | Observation matrix (Output matrix). |
| $\boldsymbol{C}$ | Viscous damping matrix of a second-order continuous-time system. |

$\boldsymbol{C}(q)$ Moving-average matrix polynomial (Innovation formulation).
$\boldsymbol{C}_i$ $i$th moving average coefficient matrix.
$\boldsymbol{D}$ Direct term matrix
$\boldsymbol{F}$ Continuous-time state matrix
$\boldsymbol{h}(\cdot)$ Impulse response function
$\boldsymbol{H}(\cdot)$ Transfer function
$\boldsymbol{I}$ Identity matrix
$\boldsymbol{K}$ Stiffness matrix of a second-order continuous-time system
$\boldsymbol{K}$ Kalman gain matrix
$\boldsymbol{M}$ Diagonal mass matrix of a second-order continuous-time system
$\boldsymbol{Q}_r$ State space Reachability matrix
$\boldsymbol{Q}_o$ State space Observability matrix

**Modal Parameters**

$f_i$ Undamped natural eigenfrequency of the $i$th mode.
$\omega_i$ Undamped natural eigenfrequency of the $i$th mode.
$\zeta_i$ Damping ratio of the $i$th eigenvalue.
$\lambda_i$ $i$th eigenvalue of a continuous-time system.
$\mu_i$ $i$th eigenvalue of a discrete-time system.
$m_i$ Modal mass corresponding to the $i$th eigenvalue.
$\boldsymbol{\Phi}_i$ Scaled mode shape corresponding to the $i$th eigenvalue.
$\boldsymbol{\psi}_i$ Eigenvector corresponding to the $i$th eigenvalue.
$\boldsymbol{\Psi}$ Modal matrix (Matrix of eigenvectors).
$\boldsymbol{R}_i$ Residue corresponding to the $i$th eigenvalue.

**System Identification**

$\theta$ Model parameter vector.
$\hat{\theta}_N$ Vector of estimated model parameters based on $N$ samples.
$\theta_0$ True model parameter vector.
$\varphi(\cdot)$ Regression vector formed by lagged measurements.
$\phi(\cdot)$ Matrix formed by $\varphi(\cdot) \otimes \boldsymbol{I}$.
$\boldsymbol{P}(\hat{\theta}_N)$ Covariance matrix of the estimated parameter vector $\theta$

**Statistical Parameters**

$\boldsymbol{S}(\omega)$ Spectral density of an output process
$\boldsymbol{u}(t_k)$ Discrete-time Gaussian distributed input process.
$\boldsymbol{e}(t_k)$ Discrete-time Gaussian distributed innovation process.
$\boldsymbol{x}(t_k)$ State vector.
$\boldsymbol{y}(t_k)$ System output (system response).
$\boldsymbol{\Sigma}(n)$ Covariance function at lag number $n$ of a discrete-time output process $\boldsymbol{y}(t_k)$.
$\boldsymbol{\Gamma}(n)$ Covariance function at lag number $n$ of a continuous-time output process $\boldsymbol{y}(t_k)$.
$\boldsymbol{\Delta}$ Covariance matrix of a Gaussian white noise input process $\boldsymbol{u}(t_k)$.
$\boldsymbol{\Lambda}$ Covariance matrix of an innovation white noise process $\boldsymbol{e}(t_k)$.

| | |
|---|---|
| $\boldsymbol{\Pi}$ | Steady state covariance matrix of a discrete-time state vector process $\boldsymbol{x}(t_k)$. |
| $\boldsymbol{\Gamma}$ | Steady state covariance matrix of the prediction of the state vector process $\boldsymbol{x}(t_k)$. |
| $\boldsymbol{P}$ | Steady state covariance matrix of the state prediction errors. |
| $\boldsymbol{W}$ | Continuous-time Gaussian white noise intensity matrix / Discrete-time Gaussian white noise covariance matrix. |

## Operators

| | |
|---|---|
| $Re(\cdot)$ | Real part of a complex number |
| $Im(\cdot)$ | Imaginary part of a complex number |
| $\lvert \cdot \rvert$ | Modulus (Magnitude) of a complex number |
| $col(\cdot)$ | Stacking of all columns of the argument matrix |
| $E[\cdot]$ | Expectation operator |
| $q$ | Delay operator ( $\boldsymbol{y}(t_k)q^{-n} = \boldsymbol{y}(t_{k-n})$ ) |
| $\otimes$ | Kronecker product |
| $D$ | Differential operator ( $D^n\boldsymbol{y}(t) = \frac{d^n y(t)}{dt^n}$ ) |

## Abbreviations and Acronyms

AR: Auto-Regressive
ARMA: Auto-Regressive Moving Average
ARV: Auto-Regressive Vector
ARMAV: Auto-Regressive Moving Average Vector
MDOF: Multi Degree Of Freedom
RMS: Root Mean Square
SDOF: Single Degree Of Freedom

# 1 Introduction

The past 30 years have witnessed major developments in the theory and application of linear systems, work that was heavily influenced by results of Kalman in the early 1960s, see Kalman [49] and Kalman et al. [50]. Certain essential advances have been made in understanding the algebraic and topological structure of linear dynamic systems, with much of this work originating from systems and control engineering. At the same time, these systems, especially for the univariate (scalar) output case, have been widely used to model and statistically treat data arising in signal processing. In particular, electrical engineers have developed algorithms for on-line and real-time model estimations, see Ljung et al. [73] and Young [113]. Finally, statistical time-series analysts, motivated by applications coming from a wide variety of fields, have developed theory and algorithms primarily for off-line model estimation, see e.g. Akaike [1] and Hannan [32]. The work of Box and Jenkins, see Box et al. [16], played a major part in this development. Statisticians have also developed the required asymptotic theory associated with estimation procedures, see e.g. Whittle [112].

In this chapter the basic concepts of system identification are introduced. Also introduced, are some of the applications of system identification in civil engineering. The introduced applications are modal analysis and vibration based inspection. Finally, in the end of this chapter the scope of the work of this thesis is stated.

## 1.1 System Identification

A convenient way of describing a dynamic system is by use of mathematical models. These models can either be represented in continuous time as differential equation systems or in discrete-time as difference equation systems. There are in general two ways to construct mathematical models:

☞ *Physical modelling.*
☞ *System identification.*

In physical modelling the construction of a dynamic model is based on physical knowledge and fundamental laws, such as the Newton 2. law of motion. On the other hand, if the physical knowledge about a dynamic system is limited, a model of the input / output behaviour of the system can be obtained through system identification based on calibration of a model using experimental data.

If the structure of the calibrated model is chosen without regard to physical knowledge, the calibrated model is called a black box model. If some parts of the model are based on physical knowledge, the calibrated model is called a grey box model. On the other hand, if the calibrated model is based completely on the physical laws, i.e. if it originates from a physical modelling, then the calibrated model is called a white box model.

Thus, system identification should not be thought of as a substitute of physical modelling, since identification can be based on model structures that have physical origin. Basically there are two categories of model structures:

- ☞ *Non-parametric model structures.*
- ☞ *Parametric model structures.*

In any case, physical modelling will always be linked with parametric model structures. Common to both categories of model structures is that they depend on the applied excitation, which may be one of the following

- ☞ *Instantaneous excitation.*
- ☞ *Periodic excitation.*
- ☞ *Pseudo-random periodic excitation.*
- ☞ *Stochastic excitation.*

In the case of instantaneous excitation, the system is either given an impulse or step excitation and the system is the left vibrating on its own. The excitation may or may not be measured. It is also possible to excite the system with a known periodic excitation, such as sinusoidal, or several periodic signals mixed to obtain a pseudo-random periodic excitation. Finally, as an alternative to the deterministic excitation, the excitation might also be a stationary stochastic process with either known or unknown statistical properties.

The combination of linear system theory, time series analysis and asymptotic theory is the foundation of modern system identification. System identification basically means modelling of the dynamic systems from experimental data. This general definition indicates that system identification is applicable in many different engineering fields.

Some of the applications are:

- ☞ *Analysis of dynamic biological functions, such as heart rate control and effects of drugs.*
- ☞ *Identification of industrial processes, and industrial plant control.*
- ☞ *Modelling of stock prices in economics*.
- ☞ *Performance study of automitives, ranging from aero space vehicles and automobiles to railway carriages.*
- ☞ *Identification of the dynamic properties of civil engineering structures, such as towers, dams, bridges, offshore structures.*

A common feature of all these examples is that their dynamic behaviour can be conceptually described as in figure 1.1.

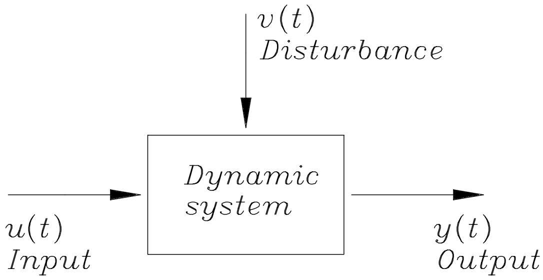


Figure 1.1: A dynamic system with input $u(t)$, output $y(t)$ and disturbance $v(t)$.

The system is driven by input $u(t)$ and affected by disturbance $v(t)$. In some cases the user can control the input $u(t)$ but not the disturbance $v(t)$. It might also be that the actual input is unknown and therefore uncontrollable in some applications. The output $y(t)$ describes how the system reacts or responds to the applied input and disturbance. Therefore, the output will be a mixture of dynamic response of the system and characteristics of the input and disturbance as well. In general, the input at previous time instances will also affect the current output. In other words, the dynamic system has memory.

### 1.1.1 Non-Parametric Model Structures

The non-parametric models are described by curves, functional relationships or tables. These analysis methods are:

- ☞ *Transient analysis.*
- ☞ *Frequency analysis.*
- ☞ *Correlation analysis.*
- ☞ *Spectral analysis.*

Transient analysis is applied when the system response is transient, i.e. generated on the basis of impulse or step excitation. The dynamic behaviour of the system is then identified on the basis of the impulse or step response. Frequency analysis is applied when the excitation is deterministic and either periodic, or pseudo-random and periodic. The measured excitation and corresponding system response is transformed to frequency domain, and the frequency response function is obtained as the ratio of the transformed response and excitation. Correlation and spectral analysis are methods that are applied to a stationary stochastically excited system. In these cases, the excitation and the system response can be characterized either by the correlation functions in time domain or the spectral densities in frequency domain. Having estimated the correlation functions of the excitation and the response the impulse response function of the system can be obtained. On the other hand, if the spectral densities of the excitation and response are estimated instead, it is possible to obtain the frequency response function.

The traditional non-parametric system identification techniques are primarily based on Fourier transform techniques, see e.g. Bendat et al. [14]. The Fourier transform techniques are elegant mathematical tools, which are ideal for theoretical analysis of dynamic systems. The Fourier transform of uniformly sampled data is usually performed by the Fast Fourier Transform (FFT). The FFT algorithms have been a key in system identification and signal processing, since the arrival in the mid 1960's, see Cooley et al. [19]. The FFT is still a popular algorithm, and is today the standard tool in commercial Fourier analysers. The reason for this popularity is the speed and reliability of the FFT algorithms. Also, the user does not have to interact to the same extent as it is necessary in system identification using parametric model structures.

However, the FFT has some limitations, which have completely escaped the attention of most practitioners. The most obvious limitation is that the FFT assumes periodic data, which is certainly not the case for sampled response from stochastically excited structures. For data records with such response, it is quite certain that:

☞ *Data records have finite length.*
☞ *Data will be non-periodic.*

In principle, the Fourier transform assumes that the amount of data is infinite. In this case the frequency response functions or the spectral densities will have an infinite frequency resolution. However, in practice the available data records have a finite length, resulting in a finite frequency resolution, see e.g. Schmidt [100].

The FFT assumes periodicity, i.e. that the finite data record of length repeats itself in both ends of the record. Since sampled stochastic signals in general exhibit non-periodicity, errors will certainly be introduced. These kinds of errors are called leakage, since the energy of the resonance frequencies leaks out. In frequency domain the effect of leakage is a seemingly higher damping of the corresponding modes. Further, in the case of closely spaced modes, it might be impossible to separate these if one of the resonance frequencies has a small amplitude compared to the other. In this case the resonance frequency with the smallest energy content may be masked completely by the resonance frequency with highest energy content.

The leakage errors are compensated by windowing the data before the FFT is applied, to secure periodicity of the data by damping the discontinuities at the ends of the data record. The problem of windowing is, that it introduces an extra damping into the system, and thus creates its own leakage problem. Leakage is a systematic error. However, there are also the random measurement errors that contaminate the data. To eliminate these random errors the data is averaged, either before or after the FFT has been applied. Several forms of averaging exist. A typical approach, though, is linear averaging of segments of the data records after the FFT has been applied to each segment, see Kay [51]. This kind of averaging will in the limit eliminate the random errors as a consequence of the central limit theorem, but not the leakage. However, for a fixed amount of data, averaging will be performed on the expense of the frequency resolution.

The finite length of the data record used in the FFT, instead of the theoretically infinite length assumed in the Fourier transform, limits the lowest frequency and the frequency resolution and causes leakage when violated. On the other hand, the finite sampling interval used in the FFT, instead of the zero interval assumed in the Fourier transform, limits the highest frequency to the Nyquist frequency and causes aliasing when violated. Aliasing superimposes the contributions of all frequencies beyond the range of zero and twice the Nyquist frequency, by folding around limits as many times as necessary. If aliasing is not taken into account, it might seriously distort any data analysis. If the sampling frequency is taken at least twice the frequency beyond which the energy is nearly zero, then the aliasing will be negligible. But for a fixed amount of data, a high sampling frequency can only be obtained at the expense of the frequency resolution. Thus, inadequate frequency resolution or leakage and aliasing are closely connected.

So in conclusion:

☞ *To reduce leakage, caused by non-periodicity of the data, the data is windowed. To reduce the noise averaging is applied, and to eliminate aliasing a high sampling frequency might be needed. All of these techniques will most certainly for a fixed amount of data result in a limited frequency resolution, and thus in inaccurate estimates of the system.*

### 1.1.2 Parametric Model Structures

Parametric models are characterized by the assumption of a mathematical model constructed from a set of parameters. These parameters are then estimated during the system identification. The mathematical model of a linear and time-invariant continuous-time system is usually in the form of a differential equation system. The equivalent discrete-time parametric model is a difference equation system. In figure 1.1 an input / output system affected by noise was shown. The appearance of the discrete-time parametric model that describes such a system depends on whether the input is measured or not. If the input is measured, then the associated parametric model will have a deterministic term as well as a stochastic term that describes the unknown disturbance. If the actual input is unknown, it is treated stochastically. In this case the description of input and disturbance will be described by a single stochastic term.

#### Model Structures using Deterministic Input

The general input / output model structure used for modelling of linear and time-invariant dynamic systems excited by deterministic input, is Auto-Regressive Moving Average with eXternal input (ARMAX)

$$\boldsymbol{y}(t) = \boldsymbol{G}(q)\boldsymbol{u}(t) + \boldsymbol{H}(q)\boldsymbol{e}(t) \tag{1.1}$$

where $\boldsymbol{G}(q)$ and $\boldsymbol{H}(q)$ are the transfer functions of the deterministic part and the stochastic part. The stochastic input $\boldsymbol{e}(t)$ are the innovations, which is an equivalent process of the noise and prediction errors. If $\boldsymbol{H}(q) = \boldsymbol{I}$ (1.1) is called an output error (OE) model. In any case the dynamic properties of the system are modelled by $\boldsymbol{G}(q)$. A parametric model structure is called multivariable when it includes several variables. If there are several outputs, it is characterized as a multivariate model structure. If it only has one output, it is termed an univariate model structure.

### Model Structures using Stochastic Input

If the input is an unmeasurable stationary stochastic process, the ARMAX model is no longer the correct model structure to use. In this case an Auto-Regressive Moving Average (ARMA) model should be applied

$$\boldsymbol{y}(t) = \boldsymbol{H}(q)\boldsymbol{e}(t) \tag{1.2}$$

The dynamical properties as well as the noise are now modelled by the same transfer function $\boldsymbol{H}(q)$. In the multivariate case the model structure is called an Auto-Regressive Moving Average Vector (ARMAV) model. As observed the choice of model structure depends on whether the input is deterministic or stochastic, i.e. whether the excitation of the structural system is known and measured or unknown. It also depends on whether the system is stationarily excited, or excited by an impulse or step excitation.

## 1.2 System Identification of Civil Engineering Structures

The early development of system theory and statistical time series analysis, i.e. the use of parametric model structures, has more or less escaped the attention of the civil engineering community. But during the sixties and seventies the need for knowledge of the modal properties of large civil engineering structures, such as high-rise buildings and bridges increased. Dynamic measurements of several high-rise buildings, suspension bridges and offshore structures were undertaken and used in system identification, see Hart et al. [35] and Jensen [43]. During this period the interest in using parametric time domain models for system identification of structural systems increased. This work was primarily motivated by Gersch et al., see [25,26,27,28]. This interest has increased ever since, see e.g. Pandit et al. [85] and Pandit et al. [87]. In offshore and civil engineering, the use of multivariate time domain models has especially attracted the attention, see e.g. Bonnecase et al. [15], Hoen [38], Pi et al. [92] and Prevosto et al. [94].

Due to the complexity of the multivariate parametric models, a lot of research on the special properties of multivariate models was performed in the early eighties, see Gevers et al. [29] and Hannan et al. [34]. This complexity and the computational effort needed to estimate these models is probably some of the reasons why the FFT-based non-parametric methods are still preferred by a lot of civil engineers.

In the field of civil engineering, system identification might be applied for several reasons. However, the following two areas have attracted much attention in the recent years

☞ *Modal analysis.*
☞ *Vibration based inspection.*

Modal analysis covers a variety of applications all based on the analysis of modal parameters. These parameters describe specific dynamic characteristics of the structure. One of the applications that uses the modal parameters as basis is vibration based inspection.

### 1.2.1 Modal Analysis

Modal analysis is based on the determination of modal parameters of a structural system. These parameters represent an optimal model, or basis, which can be used to describe the dynamics of a structural system. The modal parameters can be divided into the following four categories:

☞ *Modal frequencies*.
☞ *Modal damping*.
☞ *Modal vectors*.
☞ *Modal scaling*.

The modal frequencies are more explicitly eigenvalues, or angular or natural eigenfrequencies. Modal damping is characterized by the damping ratios, and modal vectors by the eigenvectors or mode shapes. Finally, modal masses and residues are typical parameters used to characterize modal scaling.

Since the modal parameters are directly related to the impulse and frequency response functions, as well as the correlation functions and spectral densities, they can be extracted from the non-parametric system identification methods by applying different curve fitting procedures. In case of parametric system identification methods there are direct mathematical relationships between the modal parameters and the estimated model parameters. When modal parameters are used as mathematical model of the dynamic behaviour of a system, the derived model is called a modal model. It is therefore common to use experimental modal analysis as a synonym for system identification.

### 1.2.2 Vibration Based Inspection

The accumulation of damages in a civil engineering structure will cause a change in the dynamic characteristics of the structure. The basic idea in Vibration Based Inspection (VBI) is to measure these dynamic characteristics during the lifetime of the structure and use them as a basis for identification of structural damages.

Typically, a VBI programme uses the modal parameters to describe the dynamic characteristics of a structure. A synonym for the dynamic characteristics used as basis for the VBI programme is damage indicators. In other words:

☞ *A damage indicator is a dynamic quantity, which can be used to identify the existence of damage in a structure.*

Often VBI has at random been referred to as damage detection. However, in Rytter [96] it has been put in a right perspective as a part of a VBI programme. VBI can be divided into the following four levels:

☞ *Level 1 - Detection.*
☞ *Level 2 - Localization.*
☞ *Level 3 - Assessment.*
☞ *Level 4 - Consequence.*

Methods of the first level give a qualitative indication that a structure might be damaged. Level two methods give information about the probable location of the damage as well. Methods of the third level provide information about the size of the damage, and finally the level four methods give information about the actual safety of the structure given a certain damage state. The use of a damage indicator primarily gives a qualitative indication of the existence of damage, and should therefore be characterized as a level 1 method. However, some of the damage indicators will in some cases give rough estimates for the locations of damage, which is equivalent to a primitive level 2 method.

Changes in natural eigenfrequencies are no doubt the most used damage indicators. One of the reasons for this is that the natural eigenfrequencies are rather easy to determine with a high level of accuracy. Another reason is, that they are sensitive to both global and local damages, see Rytter [96]. So comparison of estimates of natural eigenfrequencies is usually an effective level 1 method. A local damage will cause changes in the derivatives of the mode shapes at the position of the damage. This means that a mode shape having many coordinates or measurement points will be a fast way to locate the approximate position of a damage. They can therefore be characterized as a simple level 2 method. The introduction of damage in a structure will usually cause changes in the damping capacity of the structure. In Rytter [96], it has been shown that the damping ratios are extremely sensitive to the introduction of even small cracks in a cantilever beam. However, dealing with real structures, the estimation of the damping ratios of the individual modes is highly sensitive to time-varying and nonphysical sources. Thus, a satisfactory accuracy of the estimates of the damping ratios will in general be impossible to obtain. Therefore, the damping is applicable as a damage indicator, but it cannot and should not be used as the only damage indicator.

As explained, all modal parameters are in principle applicable as damage indicators. This means that they can be used at least for detection of damage, and as such be characterized as level 1 methods. However, the key to a successful VBI is the use of unbiased and low-variance modal parameter estimates as damage indicators. If the estimates are biased they might cause a false alarm, i.e. indicate a damage that does not exist. If the estimation inaccuracies are too dominant, it might be impossible to detect any significant changes. Thus, the existence of a damage might be hidden.

So in conclusion:

☞ *Successful VBI based on modal parameters requires accurate and unbiased modal parameter estimates.*

In this context the computational effort spent in obtaining reliable estimates is not so important. Further, the limitations and systematic errors of the traditional FFT-based non-parametric system identification techniques motivates the use of other techniques.

This motivation can be stated as:

☞ *The need for a more accurate estimation of the modal parameters from sampled data, compared to what traditional FFT-based non-parametric techniques can provide.*

This need is basically the reason for using the parametric models in the system identification, since the physical knowledge about a dynamic system in this way is incorporated into the system identification process.

### 1.2.3 Excitation of Civil Engineering Structures

In the case of civil engineering structures there will most likely be a natural excitation of the structure such as wind or waves. These natural forms of excitation are commonly called ambient excitation and the vibrations of the structure caused by them are called ambient vibrations. System identification of structural dynamics on the basis of ambient excitation is also referred to as ambient testing. From an experimental point of view, the simplest approach to measure the dynamic parameters of a structure is to detect the response due to ambient excitation. In the case of very large structures this approach is the only practical way of performing dynamic tests, it is simply impossible to excite such structures artificially. The ambient excitation is stochastic in nature. Therefore, it cannot be described by an explicit time-dependent function, but must be characterized by certain statistical parameters, such as its mean and covariance function. Since the structural system can be seen as a linear transformation of the applied input, this means that the response will also be stochastic, and may as such also be represented by its statistical characteristics. In Ibánes [31], Jensen [42], Rubin et al. [95] and Srinavasan et al. [104], it has been shown that ambient excitation provides a quick, inexpensive and reliable way for testing of large civil engineering structures, such as buildings and offshore structures.

An extensive survey of available literature concerning full-scale measurements on offshore platforms has been performed in Jensen [42]. It is found that the typical excitation of offshore platforms for system identification is ambient excitation. Further, in Morgan et al. [80], it has been concluded that parameter estimates obtained by ambient excitation are as good as parameter estimates obtained by external excitation. This conclusion is based on a study of several published results of ambient versus forced vibration tests of high-rise structures in the USA.

Because of the nature of dynamic testing under ambient excitation conditions, this method has advantages over others, such as the impulse and periodic excitation. Ambient excitation has a broad frequency range, and thus theoretically excites all relevant modes of a structure. Also, the use of ambient excitation in dynamic testing does not disturb the normal functioning of a structure and no excitation equipment is required for ambient testing. However, the disadvantage of ambient excitation is that its characteristics cannot be controlled and measured directly.

Since ambient excitation cannot be measured directly, it can be constructed from other measurements such as the surface elevation if system identification of an offshore structure is considered. From these measurements, sea state characteristics such as significant wave height and average zero-upcrossing period, can be estimated. These characteristics can then be used as input to models, which have been developed to describe the waves either as time series or spectral densities. The connection between the theoretical description of the waves and the forces on the structure is established using a load model, which could e.g. be the Morrison equation. A more thorough discussion of this theory can be found in Sarpkaya et al. [99]. In the case where the ambient excitation is generated by fluctuating wind pressure forces, numerous measurement projects have shown that the fluctuations may be described by a stationary ergodic Gaussian stochastic process with regard to short-term conditions, see e.g. Vickery et al. [109].

## 1.3 Scope of the Work

From the above the following can be stated concerning system identification of civil engineering structures:

☞ *The dynamic behaviour of a civil engineering structure is usually modelled by a linear and time-invariant model.*

☞ *The excitation of civil engineering structures is typically unknown ambient excitation.*

☞ *If this unknown ambient excitation is e.g. the wind, it is often modelled as a stationary Gaussian stochastic process.*

☞ *For applications such as VBI a high degree of estimation accuracy of the modal parameters is required.*

☞ *Adequate parametric model structures are not limited by frequency resolution, and can as such be more accurate than FFT-based non-parametric model structures.*

These statements imply that the parametric models of section 1.1.2 are applicable to system identification of civil engineering structures when a high degree of accuracy is needed.

Therefore, the main aim of this thesis is to investigate how to represent ambient excited civil engineering structural systems by stochastic time-domain models, and how to estimate these on the basis of sampled structural response data.

Since measurements of the true ambient excitation are not available, system identification using standard multivariate input / output ARMAX model cannot be applied. The focus is therefore put on the use of stochastic models of the Auto-Regressive Moving Average Vector (ARMAV) type. These models will be shown to be directly related to stochastic state space systems. Particular emphasis will be put on computationally accurate system identification methods, since the intention is to provide a more accurate alternative to the traditional non-parametric system identification methods typically applied in the field of civil engineering.

A secondary purpose of this thesis is to make the theory of system identification using stochastic time domain models more accessible to civil engineers. Since system identification is a relatively new field for civil engineers, it is natural to search for applicable theory and methods within disciplines that are at research front, such as automatic control engineering, mathematical system theory, econometrics, and aerospace engineering. This use of other mathematical frameworks might create conflicts with usual mathematical framework adopted in civil engineering, which means that compromises have to be made

It will be shown how an ARMAV model equivalent to the continuous-time mathematical model of a stochastically excited structural system arises. In this context, an explanation of the purpose of the moving average is emphasized. It is also shown how to account for the presence of disturbance. It will also be shown how ARMAV models are directly related to the so-called stochastic state space systems. The significant difference between the two representations is that in state space the internal structure of a system is described, whereas ARMAV models only describe the input / output behaviour of the system. A state space system is therefore referred to as an internal representation of a system, and the ARMAV model as an external representation of it. One may ask, why it is necessary to introduce two equivalent representations of a discrete-time system. For univariate systems, the ARMA model is a typical choice, whereas multivariate systems typically are represented in state space in order to obtain a first-order model, see Hoen [38]. However, the choice of representation is also dependent upon the actual application. If the modal decomposition of the univariate system is desired, the natural choice of representation would be state space, since the associated eigenvalue problem is then of first order. If the covariance function of the response of a multivariate system is desired for several time-lags, a sensible choice of representation would probably be the ARMAV model, because of its recursive structure. But in any case, it is important to notice, that the both representations describe the same system.

Since modal analysis is one of the main reasons for system identification of civil engineering structures, a thorough treatment of the modal decomposition and extraction of modal parameters will be given. In addition, guidelines for estimation of the uncertainties of the estimated modal parameters will be given.

During the Ph.D. project emphasis has been put on the practical implementation of user-friendly system identification software. Since this work is an essential part of the project, this software will be described. Two experimental cases are shown. Through these the performance of the numerical algorithms are tested, and the application of modal analysis as basis for damage detection is shown.

## 1.4 Two Experimental Cases

System identification using ARMAV models and their applications in civil engineering will be illustrated in two experimental cases, which are introduced in this section. The first case concerns a simulation study, whereas in the second case the applications of system identification, modal analysis and VBI will be shown.

### 1.4.1 Simulation of a Five-Storey Building

The purpose of this experimental case is to verify the asymptotic properties of the estimated ARMAV model. However, since the estimated model parameters are of minor importance than the estimated modal parameters, the analysis will be performed on the basis of the estimated modal parameters. The analysis will show how the estimated modal parameters, their bias, and their estimated standard deviations depend on the following variables:

☞ *Length of measurement records.*
☞ *Signal-to-noise ratio.*

The choice of other variables such as the sampling interval will also influence the parameter estimates, see Kirkegaard [53], but in this analysis the sampling interval will be fixed. The analysis will be based on simulated response of a five degree-of-freedom linear system. It is intended that the modal properties of the system should match the modal properties of a typical civil engineering structure. The linear system being simulated is a model of a two-dimensional simply supported five storey frame structure. This structure is illustrated in figure 1.2. The system response to a stationary Gaussian white noise excitation $\boldsymbol{w}(t)$ is assumed to have been measured at each floor of the structure. The excitations and the displacements of all five storeys will be used in the analysis. As seen in figure 1.2, the displacement of the fifth storey will be first element of the displacement vector $\boldsymbol{y}(t)$.

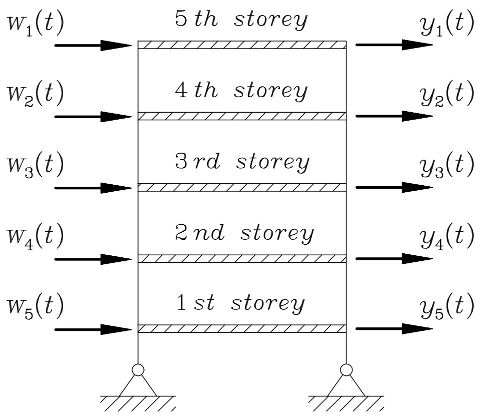


Figure 1.2: Illustration of a simply supported two-dimensional five storey frame structure.

The system will be modelled by a second-order differential equation system. The modelling, the simulations, and the statistical analysis of the results will be presented in chapter 8.

### 1.4.2 Vibration Based Inspection of a Lattice Steel Mast

The purpose of this experimental case is to illustrate the applications of system identification using ARMAV models in modal analysis and VBI. These applications will be illustrated on a lattice steel test mast. This mast is shown in figure 1.3.

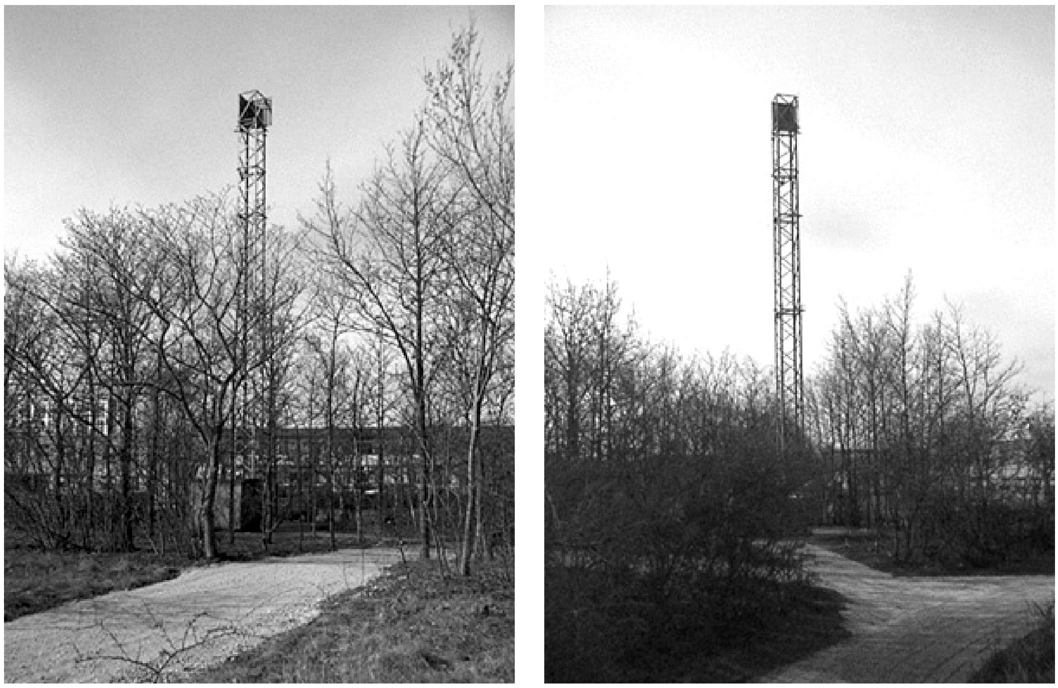

Figure 1.3: The lattice steel test mast seen from the east and north sides.

An elevation of the 20 m high steel lattice test mast seen from the west is shown in figure 1.4a. In one of the lower diagonals, which is marked in figure 1.4a, a damage has been simulated by introducing a crack and increasing its depth. The depth of the crack has been increased four times, see figure 1.4b. Before the damage is introduced, the state of the structure is referred to as the virgin state. After the introduction of the damage the four different states of the structure are referred to as damage states. The mast has been equipped with six accelerometers shown in figure 1.4a. Three of them are mounted in the top of the mast and three approximately in the middle of it. Unfortunately one of the accelerometers placed in the middle was damaged due to heavy rain, which means that the analysis has been performed with the remaining five accelerometers. Figure 1.4c show a sketch of where the accelerometers are mounted and their sensitive directions are indicated with arrows. The damaged accelerometer has the number 2.2.

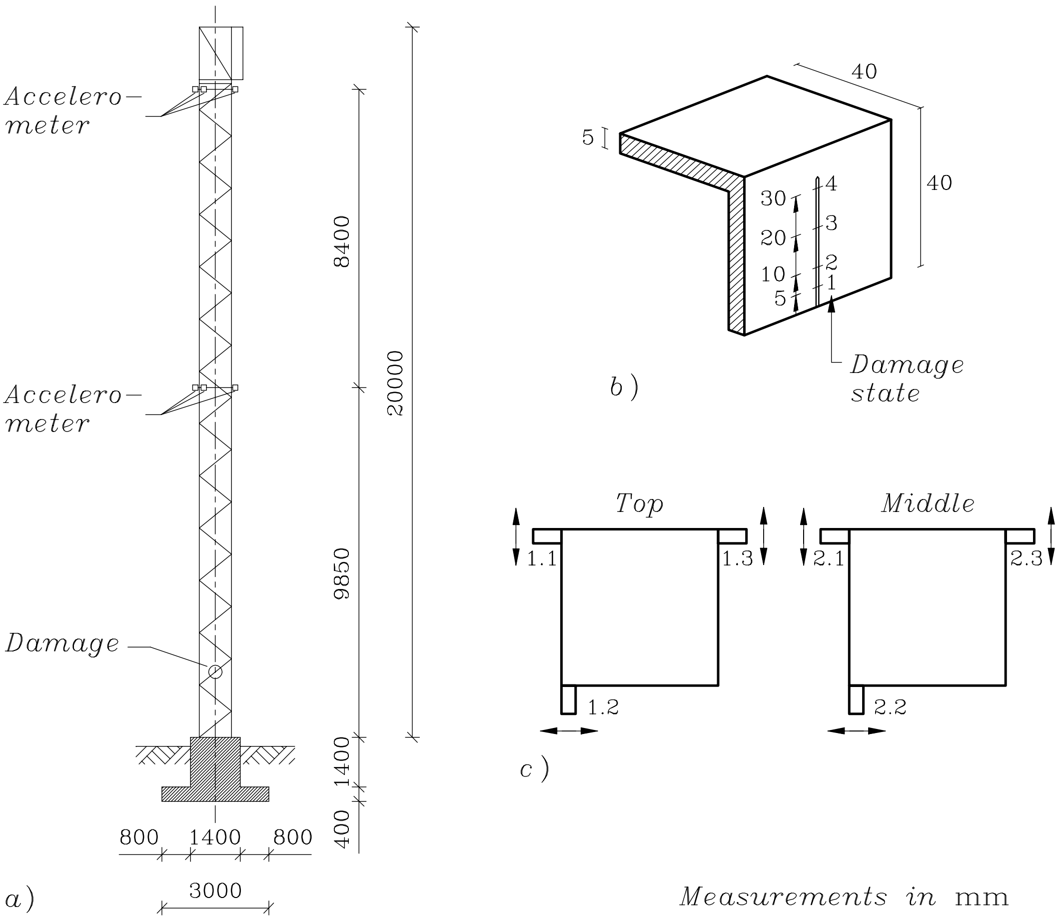


Figure 1.4: a) Elevation of the mast, where the location of the introduced damage and the six accelerometers are marked. b) The simulated damage is made by increasing the crack depth in four steps. These steps are referred to as damage states 1 to 4. c) The top and middle cross-section where the accelerometers are mounted. The sensitive directions of the accelerometers are marked with arrows. The damaged accelerometer has the number 2.2.

The four chord K-frame test mast with a 0.9 × 0.9 m cross-section is bolted with twelve bolts, three for each chord, to a concrete foundation block. This block is founded on chalk and covered by sand. The mast is constructed with welded joints. At the top of the mast there are two plywood plates in order to increase the wind forces on the structure.

The location and orientation of the accelerometers makes it possible to measure both translational and rotational vibrations. The accelerometers have the channel numbers shown in table 1.1 in the analysis in chapter 9.

| Accelerometer | 1.1 | 1.2 | 1.3 | 2.1 | 2.3 |
|---|---|---|---|---|---|
| Channel No. | 1 | 2 | 3 | 4 | 5 |

Table 1.1: Channel and accelerometer relations.

The reductions of the cross-sectional area of the diagonal due to the increase of the crack are shown in table 1.2 together with the actual crack depths.

| State of the Structure | Cross-Sectional Reduction [%] | Crack Depth [mm] |
|---|---|---|
| Virgin state | 0 | 0 |
| 1. Damage State | 7 | 5 |
| 2. Damage State | 13 | 10 |
| 3. Damage State | 27 | 20 |
| 4. Damage State | 40 | 30 |

Table 1.2: Definition of virgin and damage states.

The question is at what damage state the damage can be detected with a significant confidence. This question will be answered in chapter 9. This chapter describes the actual data acquisition and the subsequent signal processing of the data. It also describes the modal analysis made in the virgin as well as the damaged states, and how the damage is detected.

## 1.5 Reader's Guide

In this section the organization of the thesis is presented to give the reader an overview.

☞ Chapter 2 outlines the basic theory of multivariate discrete-time systems.

☞ Chapters 3-4 investigate how to represent uniformly sampled continuous-time structural systems by ARMAV models.

☞ Chapters 5-7 explain how to estimate stochastic models from sampled data and how to apply these models to modal analysis in theory and practice.

☞ Chapters 8-9 are devoted to the two experimental cases.

Since the secondary purpose of this thesis is to make the theory of system identification using stochastic time domain models more accessible to civil engineers, a brief introduction into the relevant theory of multivariate stochastic time-domain models is given in chapter 2. This chapter presents the necessary tools needed to handle stochastic state space systems and ARMAV models in an efficient manner.

In chapter 3, the modelling of stochastically excited continuous-time multivariate structural systems will be considered. First it considers a Gaussian white noise excited second-order structural system, then it is shown how this system generalizes when the excitation is nonwhite and when the number of observed outputs is different from the number of degrees of freedom of the system.

In chapter 4, the general continuous-time description of stochastically excited structural systems is discretized using a covariance equivalence approach for the system response. The result is a covariance equivalent discrete-time description. This description is either represented by a stochastic state space system or an ARMAV model.

Chapter 5 is devoted to the computational part of system identification, i.e. the procedures involved in obtaining a reasonable estimate of a chosen model structure from measured data. The chapter describes the steps in obtaining an estimate using a nonlinear Prediction Error Method (PEM) optimization approach.

Chapter 6 concerns modal analysis based on the estimated stochastic models. In this context, it is shown how to separate physical and nonphysical modes, and how to estimate the uncertainties in the estimated modal parameters.

The practical and computational aspects of system identification are considered in chapter 7. In this chapter a description of the system identification software developed during the Ph.D. project is given.

The first experimental case is given in chapter 8. The purpose of this case is to test the performance of the PEM identification algorithms through simulations. It is also the intention to verify that reliable estimates of the modal parameters and their uncertainties can be obtained.

The second case is given in chapter 9. The purpose of this case is to show how system identification and modal parameter estimation can be used as a basis for vibration based inspection. This case will only consider the first level of vibration based inspection, which concerns the detection of damage.

Finally, in chapter 10, conclusions are made, and future perspectives are discussed.

Throughout this thesis definitions are used and theorems are stated. To separate these important parts of the text from the rest, they are presented with indented text in smaller types. To indicate the end of these parts a box □ is shown. Authors will be referenced by the last name of the first author and a reference number given in brackets []. In cases where coauthor exist they will be indicated by *et al.*