

Automated Modal Parameter Estimation For Operational Modal Analysis of Large Systems

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Nomenclature

t, τ	Time
f	Frequency
$\mathbf{y}(t)$	System response
N_m	Number of measurement channels
$\boldsymbol{\varphi}, \boldsymbol{\Phi}$	Mode shape, mode shape matrix
\mathbf{C}	Covariance matrix
\mathbf{G}	Spectral density matrix
\mathbf{u}, \mathbf{U}	Singular vector, Matrix of singular vectors
$[\mathbf{s}_i]$	Diagonal matrix of singular values
d_1, d_2	Discriminator functions
Ω_1, Ω_2	Threshold levels
μ, σ, γ	Mean, standard deviation, kurtosis
$R_i^{0,j}, \mathcal{H}^{0,j}$	Correlations fixed sensors of record j and Hankel matrix
R_i^j, \mathcal{H}^j	Correlations of moving sensors of record j and Hankel matrix
$\mathcal{H}^0, \bar{\mathcal{H}}$	Merged Hankel matrices (fixed and moving)

Abstract

In this paper the problems of doing automatic modal parameter extraction and how to account for large number of data to process are considered. Two different approaches for obtaining the modal parameters automatically using OMA are presented: The Frequency Domain Decomposition (FDD) technique and a correlation-driven Stochastic Subspace Identification (SSI) technique. Special attention is given to the problem of data reduction, where many sensors are available. Finally, the techniques are demonstrated on real data.

1 Introduction

Structural Health Monitoring (SHM) is the overall name for the framework and modal parameter estimation is only one many disciplines required for a successful monitoring project. However, the general idea is to establish a baseline, or reference state and then over time compare current states with the reference state. The parameters describing the state can be different quantities but there will most likely be modal parameters among them. For a system like this to work properly, it is therefore necessary to consider how the modal parameter extraction can be made automatically. Another problem that needs to be addressed is the amount of data one need to expect. For a complex structure it may be necessary to have many measurement locations that either is measured simultaneously or bit by bit by use of reference sensors. Besides having to deal with many measurement channels it is also necessary to account for the many samples acquired in each channel. The larger the structure becomes the larger this problem gets, because the increase of necessary measurement time.

This paper will focus on these two problems; automatic modal parameter extraction and how to account for large number of data to process. Two different approaches for obtaining the modal parameters automatically using Operational Modal Analysis (OMA) will be presented.

The Frequency Domain Decomposition (FDD) technique is known as one of the most user friendly and powerful techniques for operational modal analysis of structures. However, the classical implementation of the technique requires some user interaction. In this paper an automatic algorithm for FDD is presented. This method can extract modal parameters as well as discriminate between peaks of modes and harmonics originating from forced sinusoidal excitation.

The Stochastic Subspace Identification (SSI) is a well known identification procedure well suited to handle data with non stationarity. Many variants exist. Here, we will focus on an automated version of the multipatch covariance driven subspace. This variant has the advantage over data driven versions of using less memory and merges data from different records measured at different periods of time. Special attention will be given to the problem of automated extraction of the modes. Finally, the techniques will be demonstrated on real data.

2 The Frequency Domain Decomposition Technique

When dealing with large system with many channels and many samples, the frequency domain is a desirable domain to work with. The estimation of spectral densities can easily be performed because only small segments of data will stay in the computers memory at the same time. Also the estimation time becomes an insignificant problem, due to the extremely fast implementations of the Fast Fourier Transform (FFT) available for various PC processors.

2.1 The basic algorithm

Though it is still popular to works directly with spectral densities it is cumbersome to deal with all the auto- and cross spectral densities, and the accuracy of the modal parameter estimates extracted will depend very much on how well-separated the modes are. The Frequency Domain Decomposition (FDD) technique is a way to solve these two problems, Brincker et al [1], [2]. The technique simplifies the user interaction because the user has only to consider one frequency domain function - the singular value diagram of the spectral density matrix. This diagram concentrates information from all spectral density functions. Further, if some simple assumptions are fulfilled, the technique directly provides a modal decomposition of the vibration information, and the modal information for each mode can be extracted easily and accurately. The technique works even in the case of closely spaced modes and when a lot of noise is present.

The principle in the FDD technique is easiest illustrated by realizing that any response can be written in modal coordinates

$$\mathbf{y}(t) = \boldsymbol{\Phi}_1 q_1(t) + \boldsymbol{\Phi}_2 q_2(t) + \dots = \boldsymbol{\Phi} \mathbf{q}(t) \quad (1)$$

Now obtaining the covariance matrix of the responses

$$\mathbf{C}_{yy}(\tau) = E\{\mathbf{y}(t+\tau)\mathbf{y}(t)^T\} \quad (2)$$

and using equation (1) leads to

$$\begin{aligned} \mathbf{C}_{yy}(\tau) &= E\{\boldsymbol{\Phi} \mathbf{q}(t+\tau) \mathbf{q}(t)^H \boldsymbol{\Phi}^H\} \\ &= \boldsymbol{\Phi} \mathbf{C}_{qq}(\tau) \boldsymbol{\Phi}^H \end{aligned} \quad (3)$$

Expressing that the covariance of the measurements is related to the covariance of the modal coordinates through the mode shape matrix. The H is the Hermitian transposed operator. The equivalent relation in frequency domain is obtained by taking the Fourier transform

$$\mathbf{G}_{yy}(f) = \boldsymbol{\Phi} \mathbf{G}_{qq}(f) \boldsymbol{\Phi}^H \quad (4)$$

Thus if the modal coordinates are uncorrelated, the power spectral density matrix $G_{qq}(f)$ of the modal co-ordinates is diagonal, and thus, if the mode shapes are orthogonal, then equation (4) is a singular value decomposition (SVD) of the response spectral matrix.

Therefore, FDD is based on taking the SVD of the spectral density matrix

$$\mathbf{G}_{yy}(f) = \mathbf{U}(f) [\mathbf{s}_i] \mathbf{U}(f)^H \quad (5)$$

The matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \Lambda]$ is a matrix of singular vectors and the matrix $[\mathbf{s}_i]$ is a diagonal matrix of singular values. As it appears from this explanation, plotting the singular values of the spectral density matrix will provide an overlaid plot of the auto spectral densities of the modal coordinates. Note here that the singular matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \Lambda]$ is a function of frequency because of the sorting process that is taking place as a part of the SVD algorithm.

2.4 Data Reduction – Projection Channels

Since the spectral density matrices typically consist of much more columns than there are modes participating at the difference frequencies, many of the columns of $G_{yy}(f)$ are linear dependent upon each other. For large channel counts there are no need to process all the columns and the following SVD of equation (5). Good and reliable estimates of the modal parameters can be obtained from a limited number of columns. The columns we choose will be called the projection channels.

In the example used in this paper five reference channels were used, and these references were placed carefully in order to ensure that all analyzed modes were present in at least one of them. We choose these five reference channels as our projection channels in this case. The quality of the choice of projection channels should be verified by looking at the SVD of the spectral densities. If the last plotted singular value forms a horizontal line over the frequency band of interest, and if the other singular values display a good mode separation, then the choice is fine. If not other and / or more projection channels should be included.

If more projection channels are needed, the channels to look for should contain as much new information as possible about the system compared to the channels already selected. This evaluation can be performed using a simple analysis of the correlation coefficients between the difference measurement channels.

2.3 The Manual Approach

A mode is identified by looking at where the first singular value has a peak, let us say at the frequency f_0 . This defines in the simplest form of the FDD technique - the peak picking version of FDD - the modal frequency. The corresponding mode shape is obtained as the corresponding first singular vector \mathbf{u}_1 in \mathbf{U} .

$$\boldsymbol{\varphi} = \mathbf{u}_1(f_0) \quad (6)$$

2.4 Some Helpful Indicators

The process of finding peaks on a function is easily automated. However, to help distinguish between the different physical modes, harmonics and noise we introduce a set of indicators in the following.

2.4.1 Modal Coherence Indicator

Suppose a peak has been identified in the first singular value. The question is now if this is a liable modal peak or is it just a noise peak. Calculating the correlation between the first singular vector at the peak – the mode shape vector at that point - and the first singular vector at neighboring points defines the discriminator function called the modal coherence

$$d_1(f_0) = \mathbf{u}_1(f)^H \mathbf{u}_1(f_0) \quad (7)$$

If the modal coherence is close to unity, then the first singular value at the neighboring point correspond to the same modal coordinate, and therefore, the same mode is dominating. This function is helpful in discriminating between points dominated by modal information and points dominated by noise. If the components of each of the vectors in equation (7) are random, then

$$E\{\mathbf{u}_1(f_0)^H \mathbf{u}_1(f)\} = 0 \quad (8)$$

and since the length is unity

$$Var\{\mathbf{u}_1(f_0)^H \mathbf{u}_1(f)\} = 1/N_m \quad (9)$$

where N_m is the number of measurement channels. Thus the more measurement channels we have the closer two points with random (non-physical) information will get to zero. A reasonable criterion for accepting the neighboring point as a point with similar physical information, and thus accepting the presence of physical information at that frequency, could be by introducing a threshold level Ω_1 and the requirement

$$d_1 \geq \Omega_1 \quad (10)$$

setting the limit Ω_1 equal to a number close to 1, say 0.8.

2.4.2 Modal Domain Indicator

Once a peak has been accepted as representing modal information, another discriminator function can be helpful in discriminating between different modes. In this case the discriminator function is defined as

$$d_2(f) = \mathbf{u}_1(f)^H \mathbf{u}_1(f_0) \quad (11)$$

Thus this discriminator function is not a function of the initial point given by the frequency f_0 , but is a function of the frequency f of the considered neighboring point. If a high correlation is present over a certain frequency range around the considered peak it means that over that frequency range only that mode is dominating and introducing a similar criterion

$$d_2 \geq \Omega_2 \quad (12)$$

defines a frequency range $[f_0 - \Delta f_1; f_0 + \Delta f_2]$ around each peak of modal dominance called the modal domain. The lower the value Ω_2 , the larger the size $\Delta f = \Delta f_1 + \Delta f_2$ of the corresponding modal domain. Again a good initial value of Ω_2 would be 0.8. See Brincker et al. [3] for a more comprehensive discussion the choices of Ω_1 and Ω_2 .

2.4.3 Harmonic Indicator

Unfortunately, it is not always only physical modes and broad banded noise that has to be dealt with when extracting modes from an operating structure. Often there will also be harmonics arising e.g. from rotating parts of the structure. A harmonic is easily confused with a modal peak if not special measures are taken to avoid mistakes. The reason is that a harmonic appear as a narrow peak in the spectral density functions, thus the peak will also be present in the singular values.

The best way to discriminate harmonics is by the statistical characteristics of the response in a narrow frequency band around a harmonic peak. It is well known that the statistical properties of a harmonic are very different from the properties of a stochastic response. Due to the central limit theorem, and the fact that in practice a structure is loaded by many stochastically independent forces, the stochastic distribution of a modal response will be close to Gaussian. Further, the distribution of a harmonic is very different from Gaussian since it has two distinctive peaks where the distribution goes to infinity; see Bendat et al. [4]. In Jacobsen et al [5] it is shown how to use the kurtosis to discriminate between modal peaks and harmonic peaks.

The kurtosis γ of a stochastic variable x provides a way of expressing how peaked or how flat the probability density function of x is. The kurtosis is defined as the fourth central moment of the stochastic variable x normalized with respect to the standard deviation σ as follows

$$\gamma(x|\mu, \sigma) = \frac{E\{(x - \mu)^4\}}{\sigma^4} \quad (13)$$

For zero-mean Gaussian distributed data with unit standard deviation Kurtosis is $\gamma = 3$, whereas it is $\gamma = 1.5$ for sinusoidal distributed data normalized to a standard deviation of 1. This fact is used in the harmonic detection technique described further in Jacobsen et al [5].

2.5 The Automatic Approach

For a search set of interest, which usually will be maximum singular values at all discrete frequencies between DC and the Nyquist frequency, the following procedure is our proposal for an automatic FDD approach:

1. Identify a peak on the first singular value line representing a maximum
2. Check if the peak is likely to be physical
3. If so, establish the modal domain
4. If not define a noise domain around the peak
5. Exclude the modal domain or noise domain from the search set
6. Continue until the search set is empty, the peak is below the predefined excitation level, or a specified number of modes has been estimated

The key point of the algorithm is point 2). As described earlier, it is essential at this point to include a criterion concerning the correlation between neighboring points as described by the modal coherence function d_l . Also it is essential to be able to distinguish between a harmonic peak and a modal peak. Additional criteria can be based on for instance the size of the modal domain being larger than a certain value or the damping estimate being below a certain value. If we are looking for a certain number of modes, we can pick the modes that have the largest modal domain, or the modes that have the largest excitation level.

For basic FDD we are satisfied when the peak is identified as a modal peak, and the corresponding mode shape vector is estimated. For the enhanced version of FDD an additional step is required. It is necessary to estimate the auto correlation function for the modal coordinate, C_{qq} , defined in equation (3), see e.g. Brincker et al [1],[2]. From this function it is possible to automatically extract the natural frequency and the damping ratio.

3 SSI Multipatch Merging

3.1 Algorithm

The multipatch subspace approach has been presented in Mevel et al. [6]. It is based on the fact that merging Hankel matrices from different measurement setups is only possible if setups share common reference sensors and the excitation is the same for all setups. If the first requirement is impossible to be avoided and is common for many identification methods, it has been shown that the second requirement can be dropped if proper normalization is applied to the Hankel matrix of each setup.

From the state space model,

$$\begin{cases} X_{k+1}^j &= F X_k^j + V_k^j, & \text{cov}(V_k^j) = Q_j \\ Y_{k,0}^{0,j} &= H^0 X_k^j & \text{(the reference)} \\ Y_k^j &= H^j X_k^j & \text{(sensor pool } n^o j) \end{cases} \quad (14)$$

one can define the following correlations between the reference sensors itself and between the moving and reference sensors for record j at time lag i , we got the matrix relation involving G_j

$$\begin{aligned} R_i^{0,j} &= \mathbf{E} Y_k^{0,j} (Y_{k-i}^{0,j})^T = H^0 F^i G_j \\ R_i^j &= \mathbf{E} Y_k^j (Y_{k-i}^{0,j})^T = H^j F^i G_j \end{aligned} \quad (15)$$

The Hankel matrix corresponding to the reference sensors of record j is

$$\begin{aligned}
\mathcal{H}^{0,j} &= \begin{pmatrix} R_0^{0,j} & R_1^{0,j} & R_2^{0,j} & \dots \\ R_1^{0,j} & R_2^{0,j} & R_3^{0,j} & \dots \\ R_2^{0,j} & R_3^{0,j} & R_4^{0,j} & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix} \\
&= \mathcal{O}(H^0, F) \mathcal{C}(F, G_j)
\end{aligned} \tag{16}$$

By juxtaposing the different reference Hankel matrices from all records, we got the merged reference Hankel matrix, built on the concatenation of the different autocorrelations, and thus we end up with the following decomposition

$$\begin{aligned}
\mathcal{H}^0 &= \mathcal{O}(H^0, F) \mathcal{C}(F, G) \\
R_i^0 &= (R_i^{0,1} \quad R_i^{0,2} \quad \dots \quad R_i^{0,J}) \\
G &= (G_1 \quad G_2 \quad \dots \quad G_J)
\end{aligned} \tag{17}$$

Finally, we obtain the different controllability matrices in the same modal basis by splitting the $\mathcal{C}(F, G)$ matrix in the same manner G is splitted. Those matrices are also involved in the decomposition of $\mathcal{H}^j = \mathcal{O}(H^j, F) \mathcal{C}(F, G_j)$, where both left and right terms are different, and thus the matrices can not be stacked without proper normalization. By renormalizing each Hankel matrix like

$$\bar{\mathcal{H}}^j = \mathcal{H}^j (\mathcal{C}^T(F, G_j) [\mathcal{C}(F, G_j) \mathcal{C}^T(F, G_j)]^{-1} \mathcal{C}(F, G_1)) . \tag{18}$$

We can now stack them and get the matrix $\bar{\mathcal{H}} = \mathcal{O}(H, F) \mathcal{C}(F, G_1)$, where $\mathcal{C}(F, G_1)$ is assumed to be the best conditioned controllability matrix. Finally, we do the SVD as usual for the subspace method.

This approach has been coupled with the usual subspace approach where identification is performed on each setup, and modes are extracted automatically. Usually subspace diagrams are noisy, but the multipatch diagram is much clearer, allowing determining the structural modes easily. By coupling both diagrams, using some automated extraction approach described below, one can extract modes and reconstruct modes and mode shapes. Future works will focus on improving the numerical efficiency of the method.

3.2 The Automatic Approach

The identification provides a stabilization diagram. This diagram must be analyzed in order to distinguish physical from computational modes. The algorithm must also provide the frequency, the damping and the mode shape.

The first step of the automated extraction procedure is searching for alignments using graph theory. Those alignments are defined neighborhood by neighborhood looking at the order of the points. We consider every point of the stabilization diagram as the vertex of a directed graph. For each point of a given order we define an edge from this point to the nearest point at an inferior order, and another edge from this point to the nearest point at a superior order.

The key of the algorithm is to define the nearest point. Each point of the stabilization diagram is associated to an order, a frequency, a damping and a mode shape. For any two points we can define 3 distances (frequency, damping, MAC). The nearest point will be the point which is the nearest for at least 2 distances. As the diagram is associated with a directed graph, it is trivial to extract connected sub graphs using graph theory. Each sub graph defines an alignment. For each alignment, all sub alignments of a given length are considered, in order to find the most linear part. From this linear part, we will try to extend it to a longer alignment. The automated extraction algorithm need only the minimum length of an alignment as input parameter. This algorithm is very fast and very robust, so that it can be used for a monitoring routine.

4 Example – Z24 Highway Bridge

The two automatic approaches have been tested on a fairly large set of real data. The data is from the Z24 Bridge of the SIMCES project, and the test case used is the one called Progressive Damage Test no. 10. This case consists of 9 setups each with 33 channels, except setup 5 having only 27 channels. Five common reference channels were used and these have been selected as the Projection Channels in the analysis. The data has been sampled at 100 Hz with a measurement time of 655 seconds, resulting in 65516 samples per channel.

4.1 Results of the Automatic Frequency Domain Decomposition Technique

The results in this section are obtained using the Automatic Modal Analysis component of the ARTEMIS *Extractor* 4.1. This component implements the automatic FDD approach described in section 2.5 using the indicators presented in section 2.4. The thresholds Ω_1 and Ω_2 have both been set to 0.8. In the figure below the modal coherence is shown in the light blue area on top of the diagram. The top of the diagram corresponds to a modal coherence of 1 and the bottom to 0. Especially around the first mode it is seen that the modal coherence is almost 1 for a fairly broad frequency range. The modal domains of the different modes are displayed in light green, and again this domain is quite large for the first mode as well.

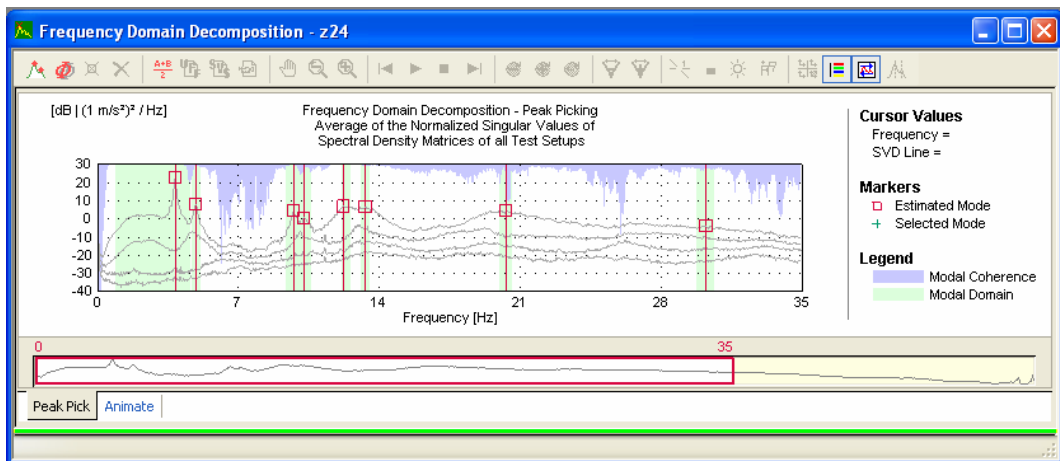


Figure 4.1: The Automatic Frequency Domain Decomposition. The red indicator shows the identified modes. The modal coherence is shown in light blue, and the modal domains in light green.

It can also be observed how the automatic algorithm is capable of detecting modes at places where the peaks are not very distinct, as with the last of the modes in the above diagram. Below, the corresponding mode shapes of all the modes are displayed:

The total processing time from starting uploading data and to the final identification of the presented modes is around 2-5 minutes depending on the PC. But even for such a large set of data, it is possible very quickly to have results of the most well-excited modes.

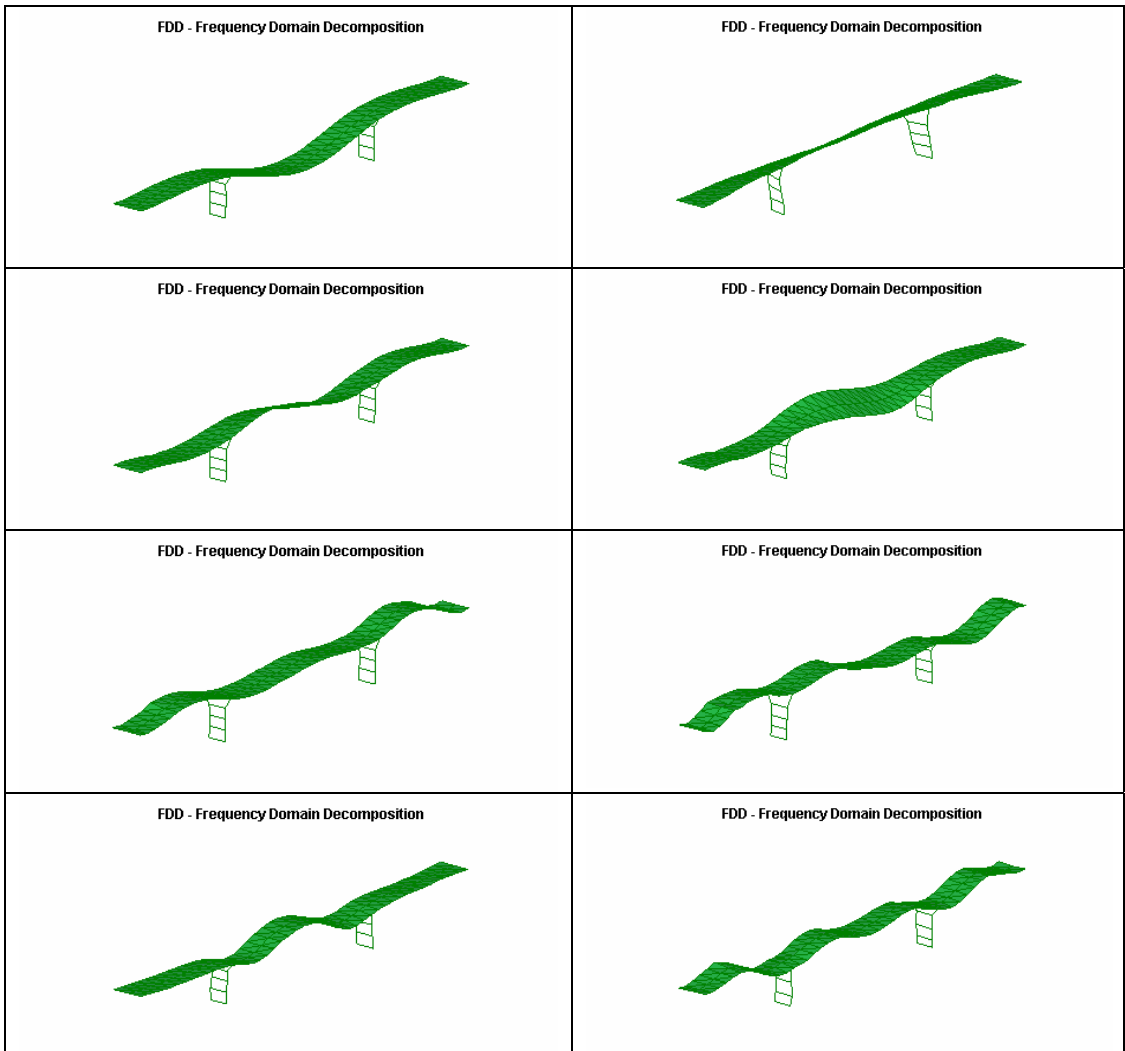


Figure 4.2: The mode shapes of the 8 identified modes.

4.2 Results of the Automatic SSI Multipatch Merging Technique

In the below figure the results is the SSI analysis are shown.

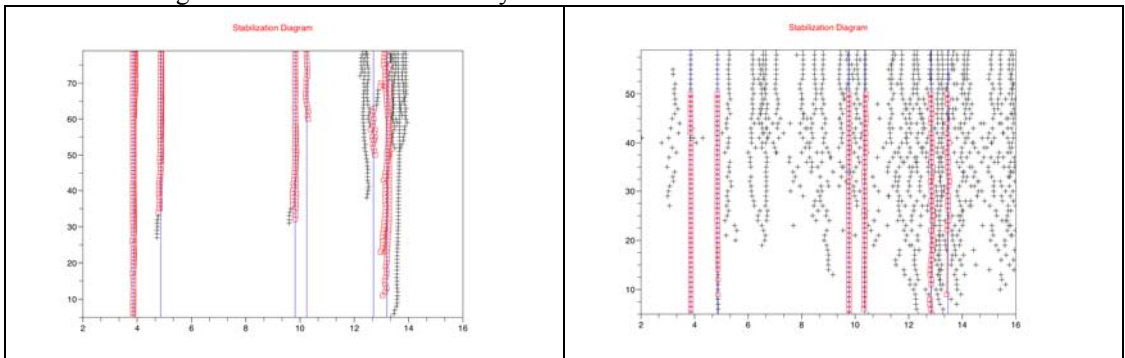


Figure 4.3: Multipatch stabilization diagram versus a traditional subspace stabilization diagram.

Figure 4.3 compares the stabilization diagram of the multipatch technique with the traditional correlation driven SSI method of one of the nine setups. The multi patch stabilization diagram is much clearer, and therefore of course much easier to work with for the automatic mode identification algorithm, that in this case identifies the six modes below 14 Hz.

5 Conclusions

The problem of doing automatic modal parameter extraction has been addressed in this paper. Two new and completely different methods have been presented.

The first is the automated Frequency Domain Decomposition technique, where the peak picking has been automated in a robust way by the introduction of modal coherence, modal domain and harmonic indicators. That it is robust refers to that the technique is able of distinguish between peaks of physical modes and peaks of non-physical contents and noise.

The second is an automated version of the multipatch covariance driven Stochastic Subspace Identification technique. The multipatch technology results in just a single stabilization diagram even when multiple setups of data are analyzed. In addition this stabilization diagram is much clearer as the only content that are the same between the different setups are the physical information of the system being measured. The noise differs from setup to setup, and is therefore suppressed in the multipatch stabilization diagrams. It has been described how an automatic mode selection can be implemented.

Finally, the two different techniques has been demonstrated on a large set of data.

6 Reference List

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