OPTIMAL DESIGN OF MEASUREMENT PROGRAMS FOR THE PARAMETER IDENTIFICATION OF DYNAMIC SYSTEMS

P.H. Kirkegaard, J.D. Sørensen & R. Brincker
Department of Building Technology and Structural Engineering
University of Aalborg
Sohngaardsholmsvej 57
DK-9000 Aalborg, Denmark

ABSTRACT

The design of a measurement program devoted to parameter identification of structural dynamic systems is considered. The design problem is formulated as an optimization problem to minimize the total expected cost that is the cost of failure and the cost of the measurement program. All the calculations are based on a priori knowledge and engineering judgement. One of the contribution of the approach is that the optimal number of sensors can be estimated. This is shown in an numerical example where the proposed approach is demonstrated. The example is concerned with design of a measurement program for estimating the modal damping parameters in a simply supported plane, vibrating beam model. Results show optimal number of sensors and their locations.

NOMENCLATURE

C: The total expected cost.

C_F: Cost of failure.

C_M: Cost of measurement program.

C_0: Cost of planning and instrumentation

C_1: Cost per sample record.

C_2: Cost of an additional sensor.

T_m: Measuring time.

P_F: Probability of failure.

$\overline{Z}$: Experiment design variables.

n_z: Number of the experiment design variables.

n: Number of random variables in $\overline{X}$.

$\overline{X}$: Random vector of correlated and non-normal variables.

$\overline{x}$: Realization of random vector $\overline{X}$.

$g(\overline{x}, \overline{p})$: Failure function.

$\overline{p}$: Deterministic parameters.

$\overline{T}$: Transformation.

$\overline{U}$: Random vector of normally distributed variables.

$\overline{u}$: Realization of random vector $\overline{U}$.

β_i: Element reliability index.

β^s: System reliability index.

$\Phi(\cdot)$: Normal distribution function.

$\Phi_m(\cdot)$: m-dimensional normal distribution function.
$\overline{\overline{\rho}}$: Correlation coefficient matrix.
$\overline{\mu}_{\overline{X}}$: Expectation of $\overline{X}$.
$\overline{\overline{C}}_{\overline{X}}$: Covariance matrix for $\overline{X}$.
$\overline{\theta}$: Vector of unknown parameters.
$n_{\overline{\theta}}$: Number of parameters in $\overline{\theta}$.
$\hat{\overline{\Theta}}$: Estimate of $\overline{\theta}$.
$\overline{\overline{C}}_{\hat{\overline{\Theta}}}$: Covariance matrix for $\hat{\overline{\Theta}}$.
J: Fisher Information Matrix.
E: Expectation operator.
$p(\overline{y}^m|\overline{\theta})$: Joint conditional probability density function of measurements $\overline{y}^m$.
N: Number of measurements.
N_s: Number of measurement points.
m: Number of significant failure modes.
$\sigma_{\hat{\overline{\Theta}}}$: Standard deviation of $\hat{\overline{\Theta}}$.
$\overline{\beta}_k^a$: Conditional reliability index vector.
$\overline{\overline{\rho}}_k^a$: Conditional correlation coefficients.
EI: Bending stiffness.
M: Mass per unit length.
C_d: Viscous damping coefficient.
t: Time parameter.
z: Space parameter.
L: Length of beam.
δ: The Dirac delta function.
δ_{ij}: The Kronecker function.
$y(z,t)$: Deflection of the beam.
$P(z,t)$: The load on the beam.
$\ddot{v}(t)$: Realization of the stochastic load process $\{\ddot{V}(t)\}$.
$q_j(t)$: Generalized coordinate.
$\phi_j(z)$: Mode shape.
ζ_i: Modal damping ratio.
D: Fatigue damage.
T_L: Expected lifetime.
σ_s: Standard deviation of stress process.
T_0: Mean period of a stress cycle.
$\Gamma(\cdot)$: Gamma function
k: Parameter in the SN-curve.
K: Parameter in the SN-curve.
$v(z_i,t)$: Realization of the stochastic measurement noise process $\{\Upsilon(z_i,t)\}$.
σ: Variance of the measurement noise

1.INTRODUCTION

The experiment design problem in dynamic system identification is to choose the experimental conditions so that the information provided by the experiment is maximized. The choice of experimental conditions for dynamic systems is known to have a significant bearing upon the achievable accuracy in parameter estimation experiments. In general, determination of the optimal experiment design, choice of the experimental conditions, leads to a highly complex optimization problem, requiring the simultaneous choice of identification algorithm, model and parameterisation, sensor type and location, actuator type and location, input excitation signal etc. Representative and excellent surveys of this area of are given in e.g. Goodwin et al. [1], Zarrop [2], Mehra [3] and Goodwin [4]. Generally, comparing different experimental designs is based on the estimator covariance matrix. Scalar functions of this estimator covariance matrix are used as experiment design criteria. In order to reduce the overall complexity of the experiment design problem, it can be assumed that the choice of identification algorithm is restricted to the class of efficient estimators, e.g. the maximum likelihood estimator. This uncouples the choice of identification algorithm from the overall experiment design since for any efficient estimator the covariance of the parameter estimates is a minimum. This minimum covariance can be estimated in terms of the Cramer-Rao lower bound or equivalently the inverse of the Fisher Information Matrix, see e.g Goodwin et al. [1].

However, when designing a measurement program the financial cost of the measurement program also has to be taken into account. The acquisition of additional information, such as performing a full-scale measuring of a structure will of course require the time, energy, and financial resources. The increased cost for this new information should be included or reflected in design of a measurement program. The increased cost may be justified if it eliminates a significant part of the uncertainty, thus leading to a lower expected probability of failure of the structure.

In this paper a method to determine an optimal measurement program devoted to parameter identification of structural dynamic systems is formulated. The problem is formulated as an optimization problem where the objective function is the total expected costs that is the costs of failure and costs of the measurement program. The cost function is introduced to make a trade-off between benefit of the new information achieved from the experiment and the costs of the measurement program. One of the main contributions of the method is that the optimal number of sensors can be estimated. The method is especially developed for dynamically sensitive structures where the reliability of the structural system is sensitive to the dynamic parameters. In section 2 the optimization problem is formulated where structural reliability theory is briefly presented since the formulation is based on reliability methods. The connection between the design variables and the total expected cost due to failure is established using modern reliability methods. Next, in section 3, the calculation procedures are presented and finally, in section 4, an example is given using the proposed method. The example is concerned with optimal design of a measurement program for optimal identification of the damping parameters in a vibrating beam. The design variables are number of sensors and location of sensors.

2. RELIABILITY BASED DESIGN OF A MEASUREMENT PROGRAM

2.1 Optimization Problem

In order to design an optimal measurement program it is suggested to minimize the total expected cost including cost of failure and the cost of the measurement program. The optimization problem of an optimal measurement program is formulated as

$$\min \quad C(\overline{Z}) = C_F P_F(\overline{Z}) + C_M(\overline{Z}) \tag{1}$$

$$s.t \quad Z_i^l \leq Z_i \leq Z_i^u \qquad i = 1, 2, ..., N_z \tag{2}$$

where $\overline{Z}$ is a vector containing N_z design variables, e.g sampling rate, number of sensors, location of sensors etc. C_F is the the cost of failure and C_M is the cost of the measurement program. The expected total cost C is the objective function . P_F is the updated probability of failure after the measurements have been performed. As constraints upper and lower limits on the design variables $\overline{Z}$ are given.

2.2 Modelling of the Cost Function

One of the difficulties with the above optimization problem is how C_F and C_M may be modelled.

When a structure fails it is necessary to pay various costs such as repair costs, reconstruction costs, clean-up costs, loss of income, costs due to loss of social prestige and possible deaths. The total cost of failure C_F may range from e.g. 2 to 5 times the initial cost of a structure, see e.g. Marshall [5].

The costs of obtaining the new information C_M is to cover not only the sample records but also the cost of statistical analysis of the information and an appropriate share of costs of planning. A simple and useful function for the cost of a measurement program is $C_M = C_0 + C_1 T_m + C_2 N$, see Ang et al. [6] . C_0 may be interpreted as representing the cost of the instrumentation and planning. C_1 may be interpreted as an additional cost per sample record with the length T_m . C_2 is the cost of an additional sensor. In some cases a more complicated cost function can be used, e.g. when a learning effect is introduced in the statistical analysis.

2.3 Structural Reliability Theory

The probability of failure P_F in (1) is estimated using the first-order reliability methods (FORM). First order reliability methods have been extensively applied in the last decade, where considerable progress has been made in the area of structural reliability theory, see e.g. Madsen et al.[7] .

A reliability analysis is based on a reliability model of the structural system. The elements in the reliability model are failure elements, modelling potential failure modes of the structural system, e.g. fatigue failure of a weld. Each failure element is described by a failure function $g(\overline{x}, \overline{p}) = 0$ in terms of a realization $\overline{x}$ of a random vector $\overline{X} = (X_1, X_2, .., X_n)$, and deterministic parameters $\overline{p}$, i.e. deterministic design parameters and parameters describing the stochastic variables, (expected value and standard deviation). $\overline{X}$ is assumed to contain n stochastic variables, e.g. variables describing the loads, strength, geometry, model uncertainty etc. Realizations $\overline{x}$ of $\overline{X}$, where $g(\overline{x}, \overline{p}) \leq 0$ correspond to failure states in the n-dimensional basic variable space, while $g(\overline{x}, \overline{p}) > 0$ correspond to safe states.

In first-order reliability methods (FORM) a transformation $\overline{T}$ of the generally correlated and non-normally distributed variables $\overline{X}$ into standardized, normally distributed variables $\overline{U} = (U_1, U_2, .., U_n)$ is defined. Let $\overline{U} = \overline{T}^{-1}(\overline{X}, \overline{p})$. In the $\overline{u}$-space the reliability index β_i is defined as

$$\beta_i = \min_{g(\overline{T}(\overline{u}), \overline{p}) = 0} (\overline{u}^T \overline{u})^{\frac{1}{2}} \tag{3}$$

If the whole structural system is modelled, as a series system, by m failure elements, and failure of the system is defined as failure of one failure element, then a generalized systems reliability index β^s of this series system can be estimated from, see e.g. Madsen et al. [7]

$$\beta^s = -\Phi^{-1}(1 - \Phi_m(\overline{\beta}; \overline{\overline{\rho}})) \tag{4}$$

where $\Phi(\cdot)$ and $\Phi_m(\cdot)$ are the normal distribution function and the m-dimensional normal distribution function, respectively. $\overline{\beta} = (\beta_1, \beta_2, .., \beta_m)$ are the reliability indices of the m most significant failure elements determined by the FORM analysis. The elements in the correlation coefficient matrix $\overline{\overline{\rho}}$ are determined in the FORM analysis. The probability of failure is

$$P_F = \Phi(-\beta^s) \tag{5}$$

2.4 Estimation of Covariance Matrix

In this section we establish the connection between the probability of failure P_F and the design variables $\overline{Z}$.

Above it is shown that the probability of failure can be estimated from a system reliability index β^s based only on the first two moments, expectation $\overline{\mu}_{\overline{X}}$ and covariance matrix $\overline{\overline{C}}_{\overline{X}}$. Normally, it is assumed that the random vector $\overline{X}$ models the following four sources of uncertainty: Inherent variability, estimation error, model imperfection and human error.

Inherent variability, often called randomness, may exist in the characteristics of the structure itself or in the environment to which the structure is exposed.

Estimation error arises from the incompleteness of statistical data and our inability to accurately estimate the parameters of the probability models that describe the inherent variabilities. Model imperfection arises from our use of idealized mathematical models to describe complex phenomena. Finally, the human error uncertainty arises from errors made by engineers or operators in the design, construction or operation phases of the structure.

Inherent variability is essentially a state of nature and the resulting uncertainty may not be controlled or reduced, i.e. the uncertainty associated with inherent variability is something we have to live with. The uncertainty associated with estimation error, model imperfection and human error may be reduced through the acquisition of additional data, the use of more accurate models and implementing rigorous quality control measures in the design, construction and operation phases of a structure.

The available statistical information, objective and subjective, on relevant variables and the set of mechanical and probabilistic models and their associated error estimates constitute the state of knowledge in a reliability problem. The state of knowledge is said to be perfect when complete statistical information and perfect models are available; otherwise, the state of knowledge is said to be imperfect. Real engineering problems invariably deal with imperfect states of knowledge.

The parameters we want to estimate by a full-scale measuring are modelled by the vector $\overline{\theta}$. $\overline{\theta}$ contains $n_{\overline{\theta}}$ parameters, e.g. modal parameters. The random vector $\hat{\overline{\Theta}}$ is an estimate of the parameter vector $\overline{\theta}$. $\hat{\overline{\Theta}}$ is included in $\overline{X}$. In this paper we only consider the statistical uncertainty of the parameter estimates which has to be expected from an experiment with the the design variables $\overline{Z}$. This means that the connection between the covariance matrix $\overline{\overline{C}}_{\hat{\overline{\Theta}}}$ for $\hat{\overline{\Theta}}$ due to estimation error and the design variables $\overline{Z}$ has to be established. The covariance matrix $\overline{\overline{C}}_{\hat{\overline{\Theta}}}$ is a function depending on the estimator assumed to be used in the experiment. Here, it is assumed that the choice of identification algorithm is restricted to the class of efficient estimators. These estimators have minimum covariance of the parameter estimates. The covariance can be estimated in terms of the Cramer-Rao lower bound, see e.g. Goodwin et al. [1]

$$\overline{\overline{C}}_{\hat{\overline{\Theta}}} = J^{-1} \tag{6}$$

where J is the Fisher Information matrix given by

$$J = E_{\overline{Y}^m}\left\{ \left(\frac{\partial \log p(\overline{y}^m|\overline{\theta})}{\partial \overline{\theta}} \right)^T \left(\frac{\partial \log p(\overline{y}^m|\overline{\theta})}{\partial \overline{\theta}} \right) \right\} \tag{7}$$

and where $\log p(\overline{y}^m|\overline{\theta})$ is the joint conditional probability density function of the N measurements

$$\overline{y}^m = \{\overline{y}^m(t_k), k = 1, 2, .., N\} \tag{8}$$

$\overline{y}^m(t)$ is an N_s-dimensional output measurement vector which is a realization of a stochastic process $\{\overline{Y}^m(t)\}$. N_s is the number of measurement points.

2.5 Calculation Procedures

Equations (1)-(7) provide the basis for designing a measurement program. The calculation procedure is as follows:

1) Estimate the covariance matrix (7) based on a structural model, a priori knowledge of data properties, engineering judgement, experimental design variables and a best prior mean estimate of $\hat{\overline{\Theta}}$
2) Calculate β^s from (4) based on the structural model, a priori knowledge of data properties, engineering judgement, experimental design variables and a best prior mean estimate and the estimated covariance matrix for $\hat{\overline{\Theta}}$
3) Calculate the total expected cost (1).
4) Determine a better estimate of the design variables.
5) Repeat 2), 3) and 4 to achieve convergence.
6) Make a sensitivity study of the measurement program design for various values of the prior mean estimate of $\hat{\overline{\Theta}}$. (This point will not be performed in the example.).

The reliability calculations in this paper are performed with the computer program PRADSS, see Sørensen [8].

The non-linear optimization problem (1) - (2) can be solved using any general non-linear optimization algorithm. In this paper the optimization problems are solved using the NLPQL algorithm, see Schittkowski [9]. The NLPQL algorithm is a effective method where each iteration consists of two steps. The first step is a determination of the search direction. The second step is a line search. Since the estimation of the system reliability index is very time- consuming it can be convenient to reduce the number of objective function calls. This can be done if instead of NLPQL another optimization algorithm is used which converges faster in the line search.

The number of function calls can also be reduced if the gradient which NLPQL requires is estimated semi-analytical and not numerical. The derivative of the objective function C with respect to a design variable Z_i is

$$\frac{\partial C}{\partial Z_i} \approx \varphi(-\beta^s)\frac{\partial(-\beta^s)}{\partial Z_i}C_F + \frac{\partial C_M}{\partial Z_i} \tag{9}$$

where $\varphi(\cdot)$ is the standard normal density function. The last term in (9) is easy to estimate analytically. The derivative of the system reliability is

$$\frac{\partial \beta^s}{\partial Z_i} = \sum_{j=1}^{n_{\overline{\theta}}} \frac{\partial \beta^s}{\partial \sigma_{\hat{\Theta}_j}} \frac{\partial \sigma_{\hat{\Theta}_j}}{\partial Z_i} \tag{10}$$

where $\sigma_{\hat{\Theta}_j}$ is the standard deviation of $\hat{\Theta}_j$. The derivative $\frac{\sigma_{\hat{\Theta}_j}}{\partial Z_i}$ can be estimated numerically. The derivative $\frac{\partial \beta^s}{\partial \sigma_{\hat{\Theta}_j}}$ follows from (4)

$$\frac{\partial \beta^s}{\partial \sigma_{\hat{\Theta}_j}} \approx \frac{1}{\varphi(\beta^s)} \sum_{k=1}^{m} \Phi_{m-1}(\overline{\beta}_k^a; \overline{\overline{\rho}}_k^a)\varphi(\beta_k)\frac{\partial \beta_k}{\partial \sigma_{\hat{\Theta}_j}} \tag{11}$$

where the correlation coefficient terms are neglected. It should be mentioned that convergence problems can be expected in optimization problems by neglecting the correlation coefficient terms. In (11) it is assumed that the m significant failure modes are numbered $1, 2, ..., m$. $\overline{\beta}_i^a$ and $\overline{\overline{\rho}}_i^a$ are the conditional reliability indices and correlation coefficients, respectively, see Sørensen [10]. The derivative of the element reliability index is estimated from, see Madsen et al. [7]

$$\frac{\partial \beta_k}{\partial \sigma_{\hat{\Theta}_j}} = \frac{1}{\beta_k} \sum_{l=1}^{n} u_{kl}^{\star} \frac{\partial \{T_l^{-1}(\overline{x}_k^{\star}, \overline{p})\}}{\partial \sigma_{\hat{\Theta}_j}} \tag{12}$$

where $\star$ indicates values at the design point.

3. EXAMPLE

In this section, an example is given to demonstrate the proposed optimization procedure. The example is concerned with optimal sensor location for identification of the modal damping parameters in a simply supported plane, vibrating Bernoulli-Euler steel beam model, see figure 1. The design variables are number of sensors N_s and location of sensors z_i. The design variable vector is defined by

$$\overline{Z} = (N_s, z_1, z_2, .., z_{N_s})^T \tag{13}$$

where T indicates a transposed vector.

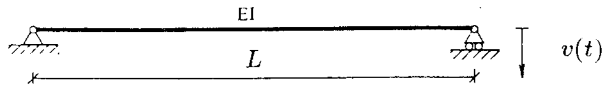

Figure 1. Bernoulli-Euler beam model.

3.1 Structural Model of Vibrating Beam

We assume that the equation of motion for the beam is given by

$$EI\frac{\partial^4 y(z,t)}{\partial z^4} + C_d\frac{\partial y(z,t)}{\partial t} + M\frac{\partial^2 y(z,t)}{\partial t^2} = P(z,t) \tag{14}$$

where $y(z,t)$ is the deflection of the beam at time t and distance z from its end. L is the beam length, M is the beam mass per unit length, C_d is the viscous damping coefficient per unit length and EI is the bending stiffness of the beam. The beam load is modelled as a motion $v(t)$ normal to the beam axis at the right base. This means that

$$P(z,t) = -\frac{z}{L}M\ddot{v}(t) \tag{15}$$

where $\ddot{v}(t)$ is a realization of a zero-mean stationary Gaussian stochastic process $\{\ddot{V}(t)\}$ with a covariance given by

$$E[\ddot{V}(t_1), \ddot{V}(t_2)] = \delta(t_1 - t_2) \tag{16}$$

where δ is the Dirac delta function. What we have assumed is that the stochastic load is white noise with variance 1.

We assume that the solution for the displacement $y(z,t)$ is

$$y(z,t) = \sum_{j=1}^{\infty} q_j(t)\phi_j(z) \tag{17}$$

where $q_j(t)$ is a generalized coordinate and $\phi_j(z)$ is the mode shape of the j'te mode. See e.g. Lin [11] for a solution for $q_j(t)$ and $\phi_j(z)$. Here, three mode shapes are taken into account. This means that the parameter vector $\overline{\theta}$ is defined by

$$\overline{\theta} = (\zeta_1, \zeta_2, \zeta_3)^T \tag{18}$$

where ζ_i is the modal damping of the ith mode and T denotes the transposed of the parameter vector. The first three flexible modal frequencies are given in Hertz as follows. 0.31. 1.23 and 2.77.

3.2 Reliability Modelling

The beam is modelled as a series system with 7 fatigue failure elements placed equidistantly. Each fatigue failure element is modelled by using the Palmgren-Miner rule in combination with SN-curves. The stress process is assumed to be zero-mean Gaussian narrow-banded. Here we don't have a narrow-banded process but the total damage is calculated as an equivalent narrow-banded damage. Then the accumulated fatigue damage D can be written, see Wirsching [12]

$$D = \frac{T_L}{T_0 K}(2\sqrt{2})^k \Gamma(1+\frac{k}{2})\sigma_s^k \quad (19)$$

where T_L is the expected lifetime. Here we use T_L=25 years. σ_s is the standard deviation of the stress process and T_0 is the mean period of a stress cycle.

$\Gamma(\cdot)$ is the gamma function. k and K are parameters in the SN- curves. Here k is modelled as a constant, k=3, and K is modelled as a random variable as $LN(6400MPa, 1024MPa)$ where LN signifies a log-normal distribution. Stress concentration is neglected. Now the fatigue failure function can be written for a given location z_i

$$g(z_i, \overline{p}, \overline{x}) = -ln(D) = -ln(T_L) + ln(T_0(z_i)) + ln(K) - kln(2\sqrt{2}) - ln(\Gamma(1+\frac{k}{2})) - kln(\sigma_s(z_i)) \quad (20)$$

The random variables in $\hat{\overline{\Theta}}$ are modelled with a log-normal distribution, mean values (0.03) and variances estimated from the estimator covariance matrix for $\hat{\overline{\Theta}}$, see next section. In the reliability calculations the K's variables for different failure elements are assumed to be uncorrelated. Each of the random variables in $\hat{\overline{\Theta}}$ is assumed fully correlated between the failure elements.

3.3 The Estimator Covariance Matrix

The estimator covariance matrix for $\hat{\overline{\Theta}}$ is now established using the Fisher Information Matrix (7).

We assume that $y(z,t)$ is directly measurable at the spatial points z_i. The observation $y^m(z_i,t)$ is described by the measuring equation

$$y^m(z_i,t) = y(z_i,t) + v(z_i,t) \quad (21)$$

where $v(z_i,t)$ denotes measurement noise at location z_i. It is assumed that the noise is a space uncorrelated stationary Gaussian white noise process $\{\Upsilon(z_i,t)\}$. The covariance is

$$E[\Upsilon(z_i,t_1), \Upsilon(z_j,t_2)] = \sigma^2 \delta_{ij} \delta(t_1 - t_2) \quad (22)$$

where

δ_{ij} and $\delta(t_1 - t_2)$ denote the Kronecker and Dirac delta functions, respectively. σ^2 is the variance of the measurement noise at the ith measurement point. Assuming the same variance at each measurement point is a usual simplifying assumption. Here we use a variance of the measurement noise corresponding to a noise to signal ratio at 0.44. The noise to signal ratio is defined as the ratio between the standard deviation of the noise and the standard deviation of the response process at 0.5 L.

Based on a set of N_s observations over $[0, T_m]$ the Fisher Information Matrix J associated with identification of $\hat{\overline{\Theta}}$ using the measurement vector in (21) is given by

$$J = \sum_{j=1}^{N_s} \frac{1}{\sigma^2} \int_0^{T_m} \left(\frac{\partial y(z_j,t)}{\partial \overline{\theta}}\right)^T \left(\frac{\partial y(z_j,t)}{\partial \overline{\theta}}\right) dt \quad (23)$$

$\frac{\partial y(z_j,t)}{\partial \overline{\theta}}$ is here estimated by numerical differentation. The response $y(z,t)$ is found from (17) based on a simulated realization of the load process $\{\ddot{V}(t)\}$. T_m is the measuring period.

3.4 Results

The optimization problem (1)-(2) is solved sequentially for varying N_s. It is assumed that the cost function can be modelled as follows

$C_0 = 10^6 \ DKK., \quad C_1 = 500 \ DKK. \quad C_2 = 10^5 \ DKK.$

C_F may vary between $10^5 - 10^9 \ DKK.$

First, in order to demonstrate the design problem (1)-(2) values of probability of failure to be expected after full-scale measurments by two sensors are shown in figure 2 as a function of the sensor location.

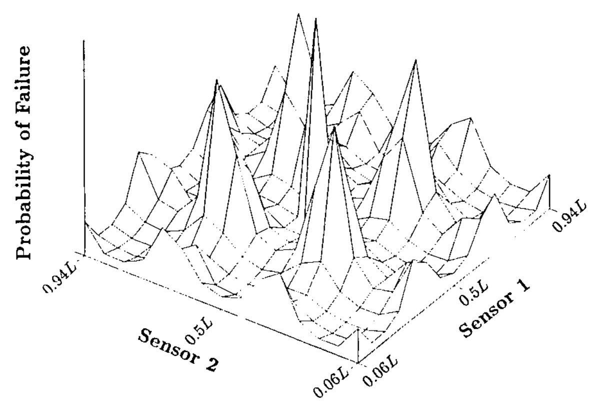

Figure 2. P_F against location of two sensors.

Figure 2. shows that our optimization problem has many local minima and a caution about local minima should be given. Therefore, the optimization problem (1)-(2) has to be solved with a range of different initial values of the design variables $\overline{Z}$. Due to symmetry of the problem we face symmetrical minima. It is also seen that the optimization problem is flat near the minima. This causes difficulties in the precise choice of optimal design on the one hand, but it also means that some imperfections in the design or in the practical positioning of sensors result in relatively small increase of error.

The optimal locations for $N_s = 1 - 5$ sensors are shown in figure 3.

N_s	z_1	z_2	z_3	z_4	z_5
1	0.168 L				
2	0.168 L	0.5 L			
3	0.168 L	0.5 L	0.5 L		
4	0.168 L	0.5 L	0.5 L	0.5 L	
5	0.168 L	0.5 L	0.5 L	0.5 L	0.5 L

Figure 3: The optimal location of sensors

The optimal sensor location 0.168 L can also be 0.832 L due to symmetry.

It is seen from figure 3 that the optimal location of sensor 2-5 is at 0.5 L. One could have expected different locations of the sensors. The reason why we don't get different locations is due to the reliability modelling on the beam model. The estimate of the system reliability depends mainly of the fatigue failure element placed at 0.5 L. For this fatigue failure element the stress process is estimated based on first and third mode. The second mode shape is zero at 0.5 L. Since the system reliability index mainly depends on the first and the third mode the measurement points are placed where the most information about the damping parameters of first and third mode can be obtained.

Figure 4 shows which values of probability of failure have to be expected after full-scale measurements have been performed with different number of optimally located sensors.

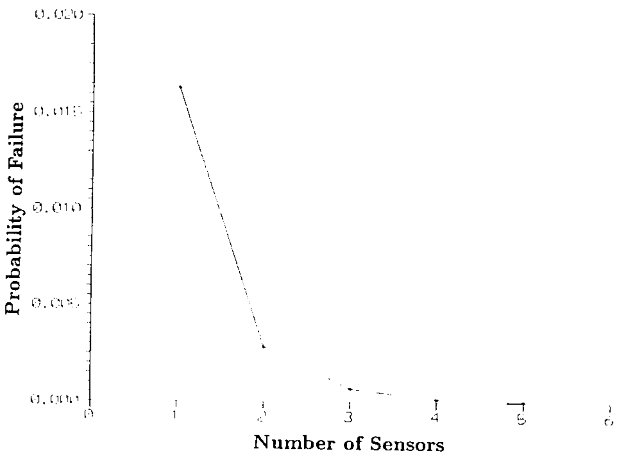

Figure 4. P_F against different number of optimally located sensors

From figure 4 one can see that the increase of the number of sensors leads to a decrease of the probability of failure. However, the decrease is small for $N_s > 2$. One can now ask, "How many sensors should be used in a full-scale measuring". The increased cost of an additional sensor may be justified if it eliminates a significant part of the uncertainty, thus leading to a lower expected probability of failure of the structure. Therefore, the increased cost of an additional sensor should be reflected in a measurement program design method as proposed in this paper

Optimal solutions of the optimization problem (1)-(2) against different number of optimally located sensors are shown in figure 5 for various cost of failure C_F.

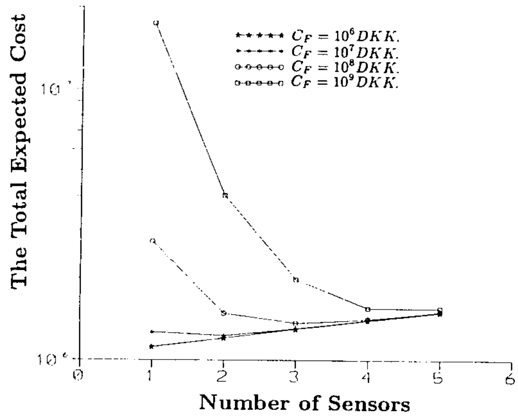

Figure 5. The total expected cost C against different number of sensors.

It is seen, as expected, that the optimal number of sensors increases when C_F increases, which means that acquisition of more information is, of course, more relevant when the cost of failure increases.

4. CONCLUSIONS

Design of an optimal measurement program is formulated as an optimization problem to minimize the total expected costs due to failure costs and cost of the measurement program. The approach is based on modern reliability theory. The calculations are based on the a priori knowledge of the data properties and engineering judgement. An example concerned with optimal sensor location for estimating the modal damping parameters in a simply supported plane, vibrating Bernoulli-Euler beam model is given. Tentative results indicate that the method works e.g to estimating optimal number of sensors and their location. However, to prove the pratical value of this approach, more complex examples should be investigated.

ACKNOWLEDGEMENT

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4. REFERENCES

[1] Goodwin,G.C. & R.L. Payne: *Dynamic System Identification: Experiment Design and Data Analysis.* Academic Press, 1977.

[2] Zarrop, M.B.: *Optimal Experiment Design for Dynamic System Identification.* Springer-Verlag, New York, 1979.

[3] Mehra, R.K.: Choice of Input Signals. In *Trends and Progress in Systems Identification* (P. Eykhoff,ed.). Pergamon Press, Elmsford, New York, 1981.

[4] Goodwin, G.C.: Identification: Experiment Design. In *Systems and Control Encyclopedia* (M. Singh,ed.). Pergamon Press, Oxford, 1987.

[5] Marshall, P.W.: *Strategy for Monitoring, Inspection and Repair for Offshore Platforms.* Structural Integrity Techn., 1979.

[6] Ang. H-S & W.H. Tang: *Probability Concepts in Engineering Planning ad Design.* Vol. 2 ,1984.

[7] Madsen, H.O, S. Krenk & N.C. Lind:*Methods of Structural Safety.* Prentice-Hall, 1986.

[8] Sørensen, J.D.:*PRADSS: Program for Reliability Analysis and Design of Structural Systems.* Structural Reliability Theory, Paper No. 36, Denmark, 1987.

[9] Schittkowski, K.:*NLPQL: A FORTRAN Subroutine Solving Constrained Non-Linear Programming Problems.* Annals of Operations Research, 1986.

[10] Sørensen, J.D.:*Reliability Based Optimization of Structural Systems.* Structural Reliability Theory, Paper No. 18, The University of Aalborg, Denmark, 1986.

[11] Lin, Y.K: *Probabilistic Analysis of Structural Dynamics.* McGraw-Hill International Book company, 1967.

[12] Wirsching, P.H: *Fatigue Reliability for Offshore Structures* . ASCE, Journal of Structural Engineering, Vol. 110, No. 10, pp.2340-2356, 1984.