# Detection of Fatigue Damage in a Steel Member

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## ABSTRACT

In this paper the possibilities of detection of crack extension in a steel beam by observation of changes in the dynamical response are investigated. Sytem changes are observed by frequency domain and time domain techniques. The position and the size of the crack by finite element calculations. The estimated values are compared to the real values observed in the experiment.

## 1 INTRODUCTION

The introduction of a crack in a structure will cause a local reduction in stiffness and an increase in the damping capacity.

The local reduction in stiffness will cause a decrease in the eigenfrequencies and a discontinuity in the mode shapes at the crack position (see figure 1).

The crack will in many cases by turn be open and closed, which implies that the stiffness of the structure becomes nonlinear. This nonlinearity can for instance be revealed by means of the response spectra of the structure, where the nonlinearity will cause subharmonics and superharmonic peaks (see e.g. Tsyfanskii et al [3]).

The increase in damping is due to the fact that the beam will have a yielding zone at the crack tip. This means that energy will be dissipated in this zone during a load cycle, which implies an increase in the damping capacity of the structure.

The above-mentioned effects of a crack means that the measurement of eigenfrequencies, mode shapes and damping ratios during the lifetime of a structure can be used for detection of cracks in structures.

The aim of this paper is to present and evaluate a method, where changes in eigenfrequencies and mode shapes are used for the detection of cracks in a civil engineering structure.

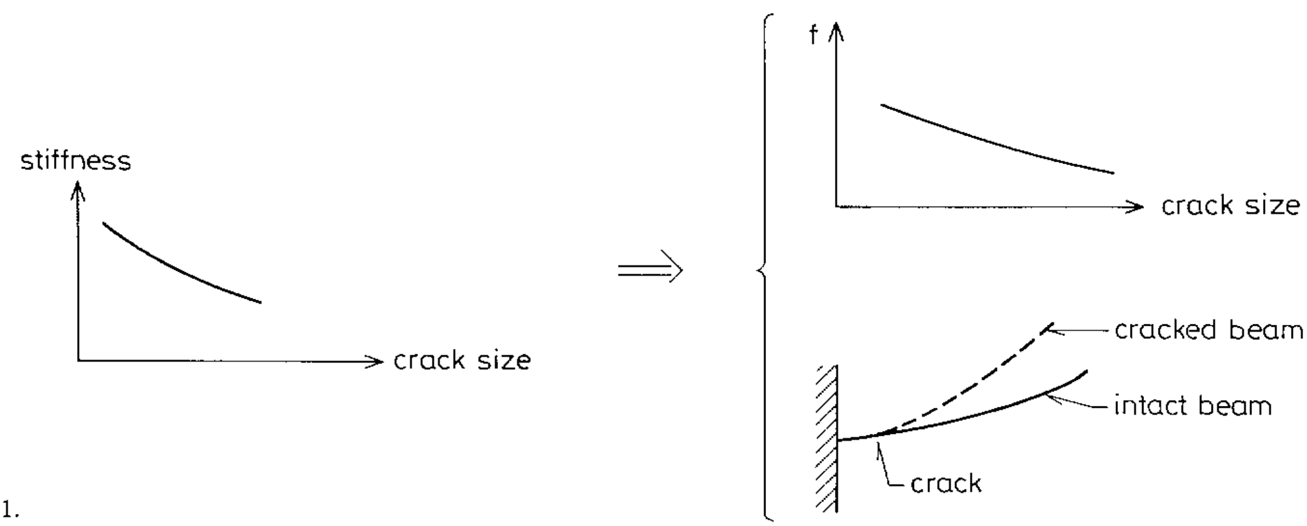



Figure 1.

Section 2 contains a presentation of the different system identification methods used during the experimental tests.

The above-mentioned decrease in stiffness is included by means of a model developed from fracture mechanics, as described in section 3. The model is included in the stiffness matrix for a finite beam element, which is used during the damage detection, as shown in section 5.

However, the use of changes in eigenfrequencies and mode shapes will be hopeless if the finite element model is not calibrated immediately after the ending of the construction work. A method based on non-linear optimization for the performance of this calibration is given in section 4.

The evaluation of the methods given in section 6 is based on results from experimental measurements on a 2 meter long hollow section cantilever, which contains a fatigue crack.

## 2 SYSTEM IDENTIFICATION TECHNIQUES

A large number of system identification methods is available (see e.g. Jensen [1]) and it is out of the scope of this paper to give a presentation of them.

The system identification methods work either in the time or in the frequency domain. The choice of domain/method is highly depending on the nature of the problem and especially on the dynamic characteristics , which have to be identified. The identification of eigenfrequencies can for instance be done with required accuracy in both domain. However, the identification of damping ratios of ligthly damped structures gives much more reliable estimates, when the identification is performed in the time domain than in the frequency domain. The latter is due to the fact that the information about the damping is spread over a long time interval in the time domain and concentrated in small area around the resonance peaks in the frequency domain (see figure 2).

The cantilever used in the experimental tests is quite ligthly damped (damping ratio $\zeta < 0.001$ ), which means that time domain system identification methods are therefore used for the identification of the modal properties of the first mode. However the eigenfrequency of the second mode is determined through a FFT-analysis.

The experimental tests include only measurement of free decays.

The identification of the eigenfrequency $f_o$, the modal damping ratio $\zeta$ and the relative modal coordinate in the measurement point of the first mode from free decays is based on a minimization for each measurement point of the errorfuntion $F(\bar{\theta})$ given in equation (1), which expressed the difference between the theoretical free decay curve and the measured.

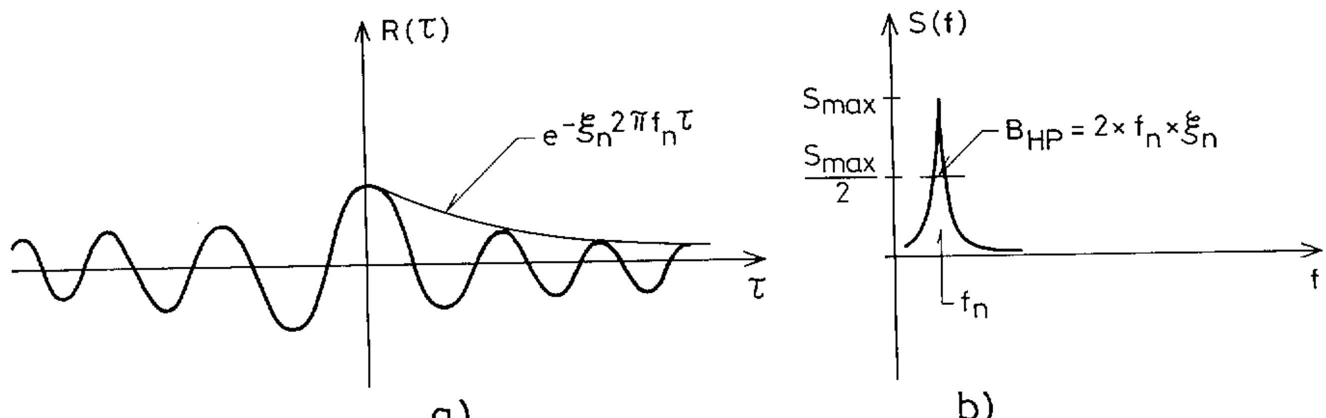
$$F(\bar{\theta}) = \sum_{j=1}^{N}(x_j - X_i e^{-2\pi\zeta f_o (j-1)/f_s} \cos(\sqrt{1-\zeta^2}2\pi f_o (i-1)/f_s + \phi))^2 \qquad (1)$$

where $x_j$ is the $j'th$ sampled value, $X_i$ is the amplitude at measurement point $i$, $f_s$ is the sampling frequency and $\phi$ is the phase.

The minimization of $F(\bar{\theta})$ is performed by mean of the M-file CONST in the MATLAB optimization toolbox [4].

The ratio between the modal coordinates of two measurement points is calculated as

$$\frac{\phi_2}{\phi_1} = \frac{X_2}{X_1} \qquad (2)$$




$f_n$ : eigenfrequency of the n'th mode.

$\zeta_n$ : modal damping ratio of the n'th mode.

Figure 2. a) Time domain, Auto correlation function. b) Frequency domain, Autospectrum.

# 3 STIFFNESS MODEL FOR A CRACKED BEAM

This section contains a presentation of an advanced model for a cracked beam section. The model is developed from fracture mechanics. The cracked section in the beam in bending modes will be modelled as a spring (see figure 3).

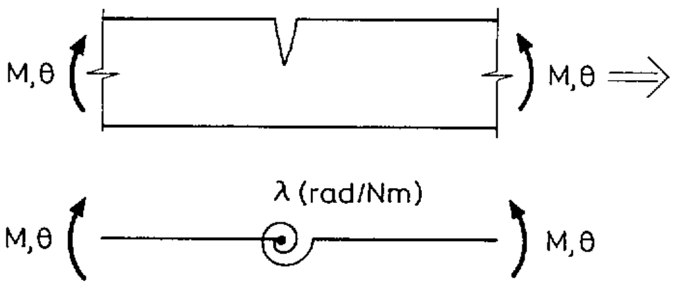



Figure 3.

The fundamental principle in the determination of the spring stiffness $\lambda$ will be summarized in the following for hollow section beam in bending (see figure 4).

The fatigue cracks are initiated from the end of a 20 mm long laser-cutted slot. The fatigue cracks are supposed to grow by the same speed at each side of the slot.

The strain energy release rate $\mathcal{G}$ is related to the stress intensity factor $K_I$ (see e.g. Hellan [6],p. 58) and to the compliance $\lambda$ of the cracked beam element (see e.g. Hellan [6],p. 55), as shown in equation (3).

$$\mathcal{G} = \frac{\beta}{E} K_I^2 = \frac{M^2}{2} \frac{d\lambda}{dA} \tag{3}$$

where $E$ is Young's modulus, $\nu$ is the Possion ratio, $M$ is the bending moment and $A$ is the crack surface. The factor $\beta$ is given by

$$\beta = \begin{cases} 1, & \text{for plane stress;} \\ 1-\nu^2, & \text{for plane strain.} \end{cases} \tag{4}$$

This means that the increase in compliance $\Delta\lambda$ caused by a crack can be calculated by

$$\Delta\lambda = \frac{2\beta t}{E} \int_0^{a_c} \left(\frac{K_I}{M}\right)^2 da \tag{4}$$

where $a_c$ is half of the total crack length and $t$ is the wall thickness of the member.

The stress intensity factor $K_I$ as a function of the crack size and beam dimension is shown in equation (6) for the crack in the flange (see Tada et al [5], p. 2.2).

$$K_I = \frac{Mh}{2I}\sqrt{\pi a}\Big(1 + 0.128(\frac{a}{b+h}) - 0.288(\frac{a}{b+h})^2 + 1.525(\frac{a}{b+h})^3 + (\frac{a}{b+h})^4\Big) \tag{6}$$

where $I$ is the moment of inertia, $b$ is the width of the profile, $h$ is the height of the profile and $a$ is half of the crack length.

Introducing the expression (6) into equation (5) and solving the integral gives.

$$\Delta\lambda = \frac{\pi\beta t h^2}{EI^2}\Big(0.5a_c^2 + 0.0853\frac{a_c^3}{b+h} - 0.1399\frac{a_c^4}{(b+h)^2} + 0.5953\frac{a_c^5}{(b+h)^3} + 0.0789\frac{a_c^6}{(b+h)^4} - 0.1255\frac{a_c^7}{(b+h)^5} + 0.2907\frac{a_c^8}{(b+h)^6}\Big) \quad for \quad 2a_c < b \tag{7}$$

The above-presented method has been used by several authors (see e.g Okamura [7] and Ju [8]). The expression used for $K_I$ varies from paper to paper, which results in small dif-

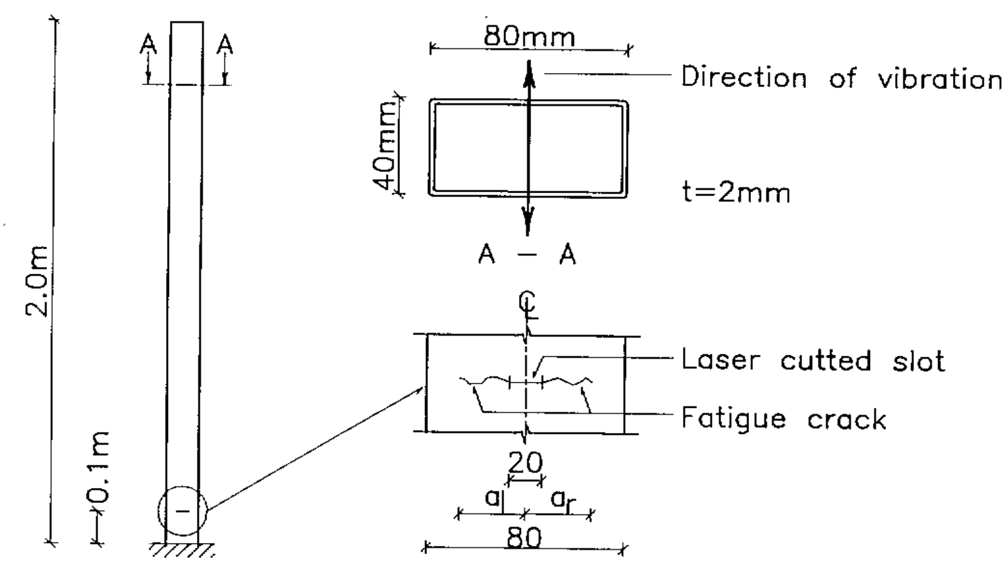



Figure 4

ferences in the results. The latter clearly demonstrates that a calibration of the diagnosis function $\mathcal{F}(\bar{R})$ is preferable (see section 5). The calibration shall be based on measurements from beams with fatigue cracks instead of saw cuts (see e.g. Chondros & Dimarogonas [9] and Cawley & Ray [10]).

The stiffness matrix for a finite beam element containing a crack has been set up and used in the diagnosing session (see section 5).

# 4 CALIBRATION OF MODELS AT VIRGIN STATE

The virgin state values for the dynamic characteristics are used for calibration of the mathematical model (typical a finite element model) to secure that the model describes the real structure in the best possible way. The calibration may shortly be written as , where $\mathcal{M}$ is an appropriate choiced functional.

$$Model = \mathcal{M}(measurement) \tag{8}$$

The calibration of finite element models is commonly performed by convergence analysis, where the number of elements is increased until convergence is obtained for the eigenfrequencies of relevance. However, an increase in the number of elements leads to more time-consuming and expensive computer runs. This is clearly undesirable, since the finite element model has to be used after each of the subsequent periodical measurements.

The calibration is in this paper performed by minimizing the function $F_V(\bar{X})$ given in equation (9) with respect to the model parameters in the vector $\bar{X}$. The vector $\bar{X}$ can for instance consists of element length, weigth of accelerometers and density.

$$F_V(\bar{X}) = \sum \left(1 - \frac{\Theta_i^C(\bar{X})}{\Theta_i^M}\right)^2 W_i \tag{9}$$

where $\bar{\Theta}^M$ contains the measured dynamic characteristics, $\bar{\Theta}^C(\bar{X})$ contains the calculated dynamic characteristics, $\bar{W}$ contains weighting parameters and $\bar{X}$ contains the model parameters.

The elements in the vector $\bar{\Theta}^M$ and $\bar{\Theta}^C$ are eigenfrequencies and mode shapes. The weighting vector $\bar{W}$ is introduced for two purposes. The first purpose is to favour the critical dynamic characteristics, which were found in the sensitivity analysis. The second purpose is to secure that the parameters in $\bar{\Theta}^M$ are weigthed with respect to how well they have been identified. The elements in the weighting vector $\bar{W}$ are in this paper taken as the reciproc value of the coefficient of variation of the single dynamic parameter.

The minimization of $F_V(\bar{X})$ is in this paper performed by means of the computer program NLPQL [2].

The number and length of elements cannot be changed later, because such changes may lead to changes in the dynamic characteristics, which are comparable to the changes due to e.g. crack growth.

# 5 DIAGNOSIS TECHNIQUES

A diagnosis of the state of damage has to be given, if a periodical measurement reveals significant changes in the dynamic characteristics. This diagnosis problem can in general be written as

$$\bar{D} = \mathcal{F}(\bar{R}) \tag{10}$$

$D$ is a damage vector, which for instance contains information about the size and location of a crack. The vector $\bar{R}$ contains information about the changes in the dynamic characteristics (e.g. eigenfrequencies) of the structure. Thus the main task in the development of a damage detection scheme based upon results from vibration measurements is to develop an expression for the function $\mathcal{F}(\bar{R})$, which gives an unambiguous $\bar{D}$ for a given $\bar{R}$.

A crack in a structure is defined by its size and location, thus the vector $\bar{D}$ contains two elements for each crack. It is therefore obvious that the vector $\bar{R}$ has to contain at least two elements for each crack to be revealed. However, the required number of elements in $\bar{R}$ will in many cases be in creased due to symmetry reasons.

The size of the vector $\bar{D}$ is of course unknown in a practical problem and can in principle be infinite, whereas the number of elements in the vector $\bar{R}$ normally will be limited by the measurement system in use. Furthermore, the elements in $\bar{R}$ will be defective due to different factors such as the signal to noise ratio and the length of the records.

The problem given in equation (10) has in this paper been solved in two steps. The loaction of the crack has been estimated first and the length of the crack afterwards.

It can be shown (see e.g. Cawley & Adams [11]) that the ratio between the changes in eigenfrequencies due to a local damage is only a function of $x_c$. Cawley and Adams defines the error in assuming the damage to be at position $x^*$, given frequency changes $\delta f_i$ and $\delta f_j$ in the modes $i$ and $j$ as

$$\begin{aligned} e_{x^*ij} &= \frac{s_{x^*i}/s_{x^*j}}{\delta f_i/\delta f_j} - 1, \qquad s_{x^*i}/s_{x^*j} \geq \delta f_i/\delta f_j \\ e_{x^*ij} &= \frac{\delta f_i/\delta f_j}{s_{x^*i}/s_{x^*j}} - 1, \qquad s_{x^*i}/s_{x^*j} < \delta f_i/\delta f_j \end{aligned} \tag{11}$$

where $s_{x^*i}$ is the sensitivity in mode $i$ against failure at $x^*$. The total error in assuming failure at position $x^*$ is

$$e_{x^*} = \sum_{all\ pairs\ i,j} e_{x^*ij} \tag{12}$$

$e_{x^*}$ has been calculated for a crack positioned succesively for each 0,01 m. The crack position $x_c$ is taken as that $x^*$, which gives the minimum value of $e_{x^*}$.

The crack length $a$ has been estimated by minimizing the function $F_P(a)$ given in equation (13) with respect to $a$.

$$F_P(a) = \sum \left( \frac{\Theta_i^M - \Theta_i^C(a)}{\Theta_i^V} \right)^2 W_i \qquad (13)$$

subjected to

$$0 \leq 2a \leq b$$

where $\bar{\Theta}^M$ contains the measured dynamic characteristics, $\bar{\Theta}^C(a)$ contains the calculated dynamic characteristics, $\bar{\Theta}^V$ contains the measured values of the dynamic characteristics at virgin state and $\bar{W}$ contains the weighting parameters.

The minimization of $F_P(a)$ is in this paper performed by means of the computer program NLPQL [2].

The purposes of the vector $\bar{W}$ are identically with those described in connection to equation (14).

## 6 EXPERIMENTAL RESULTS

The applicability of the methods described in the previous section is evaluated in this section through a detection of a fatigue crack in a cantilever (see figure 10).

The crack was initiated by a 20 mm long laser cutted slot in one of the flanges. Laser cutting was applied in order to get a cut width a small as possible and thereby minimize the errors by using a slot instead of a fatigue crack (see e.g. Cawley & Ray [14]). Crack growth was obtained by mean of a B & K vibration exciter giving a sinus-loading with a frequency nearby the first eigenfrequency of the cantilever (see figure 5).

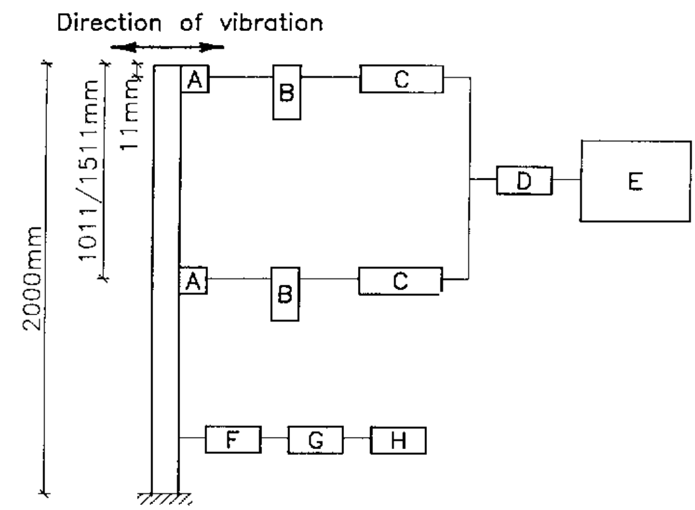
The analysis in this section are based upon measurement of the three lower eigenfrequencies and the ratio between the modal coordinate for first mode shape of the two measurement points at virgin state ( $a = 0$) and for a situation with $2a \approx 60mm$ ($\approx 3/4$ of the flange).

The results from the virgin state measurements are shown in table 1.

| | Measured value | Coef. of variation |
|---|---|---|
| $f_1$, Hz | 11.3087 | 0.0024 |
| $f_2$, Hz | 70.56 | 0.05 |
| $f_3$, Hz | 190.0 | 0.07 |
| $\phi_2/\phi_1$ | 0.100 | 0.013 |

Table 1. Results from virgin state measurements.

The finite element model of the cantilever include 8 beam elements. The minimization of the function $F_V(\bar{X})$ has been performed solely with respect to the length of the elements. The results are shown in table 2 and 3.




A: Accelerometer, B&K, type 4370
B: Charge amplifier, B&K, type 2635
C: Rockland filter, type 2582
D: A–D converter, type DT2821
E: Compaq Portable III
F: Vibration exciter, B&K 4809
G: Power amplifier, B&K 2706
H: Function generator, HP3311A

Measurement (A–E); Fatigue (F–H)

Figure 5. Instrumentation

| Element | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| Lentgh ,mm | 318 | 179 | 155 | 342 |

| Element | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Lentgh ,mm | 460 | 185 | 147 | 214 |

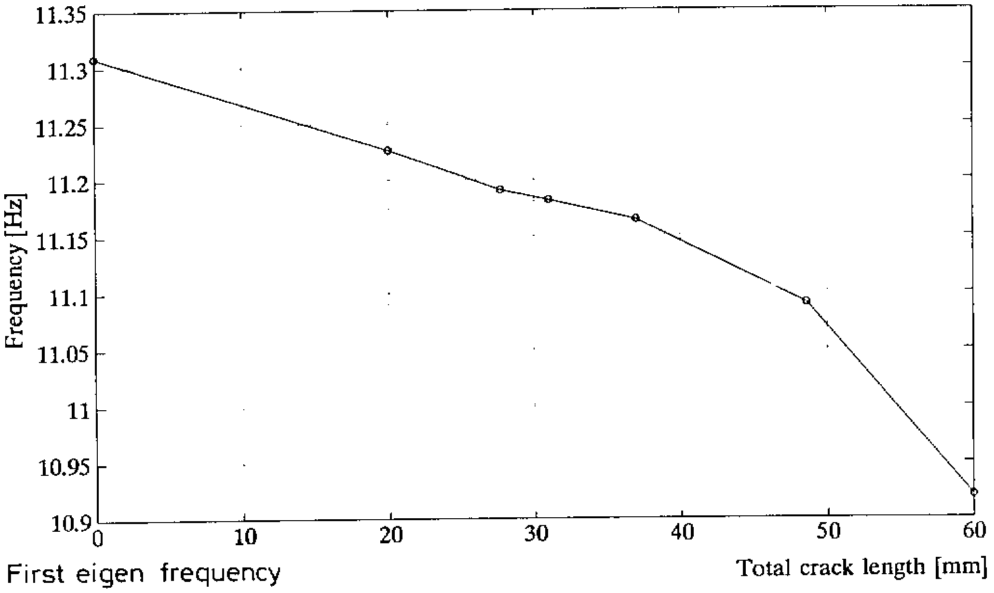
Table 2. Length of elements.

| | Measured value | FE-model |
|---|---|---|
| $f_1$, Hz | 11.3087 | 11.3103 |
| $f_2$, Hz | 70.56 | 71.21 |
| $f_3$, Hz | 190.0 | 196.1 |
| $\phi_2/\phi_1$ | 0.100 | 0.938 |

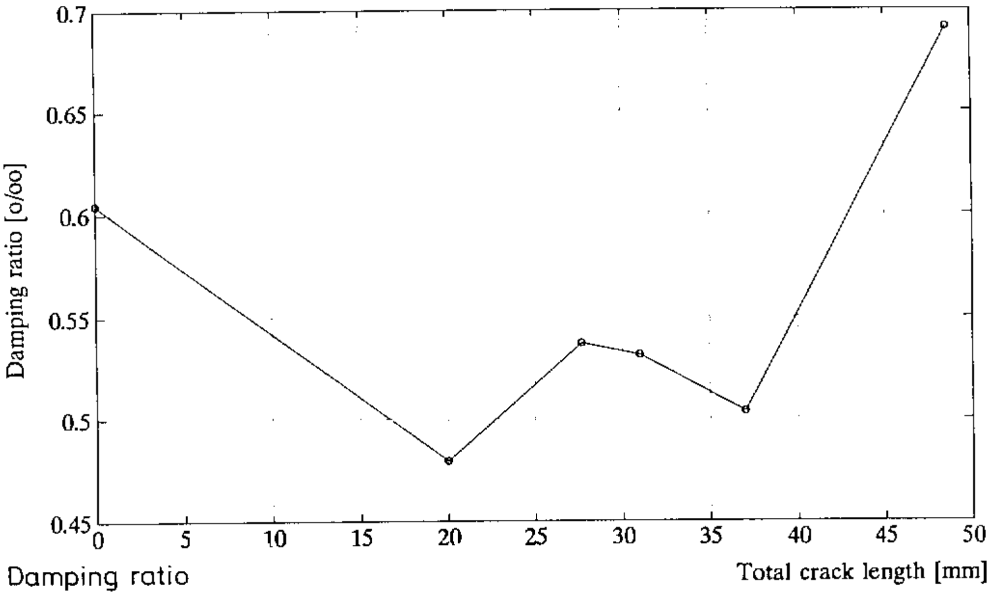
Table 3. Measured and calculated values at virgin state.

The differences between the measured values and the calculated values are probably due to the fact that the support is totally fixed in the finite element model, while some rotation is possible in the fixture of the test cantilever.

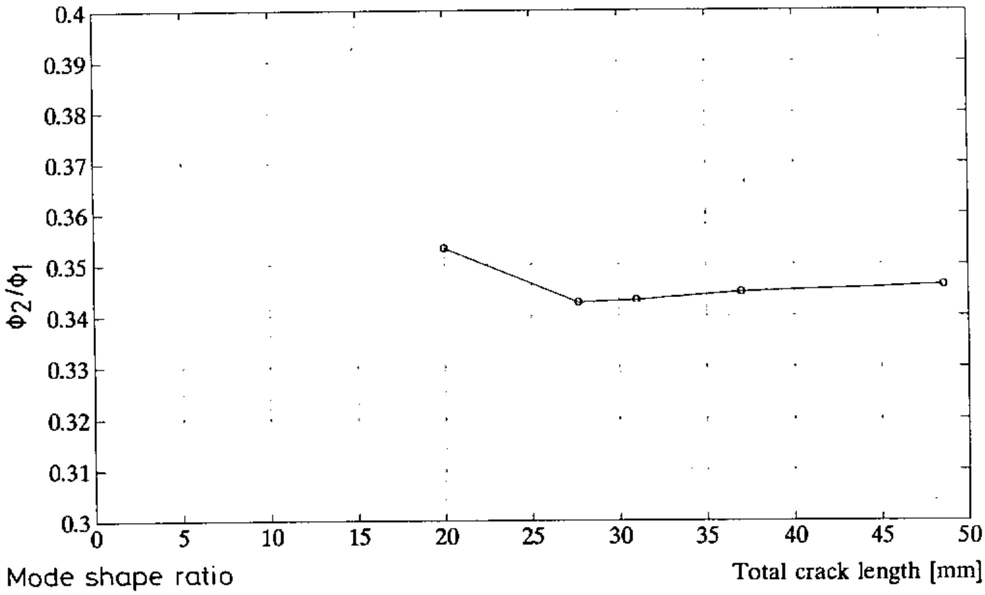
The first eigenfrequency, the damping ratio of the first mode and the ratio between the modal coordinate at the middle and at the top have been estimated for different crack sizes during the fatigue loading. The results are shown in figure 6.




a) First eigen frequency




b) Damping ratio




c) Mode shape ratio

Figure 6.

The results from the periodical measurement ($2a \approx 60mm$) are shown in table 4.

| | Measured value | Coef. of variation |
|---|---|---|
| $f_1$, Hz | 10.92 | 0.003 |
| $f_2$, Hz | 68.9 | 0.007 |
| $f_3$, Hz | 187.1 | 0.050 |
| $\phi_2/\phi_1$ | 0.100 | 0.02 |

Table 4. Results from "periodical" measurements ($2a \approx 60mm$).

The estimated values for $x_c$ and $a$ are shown in table 5.

| | Measured value | Estimated value |
|---|---|---|
| $x_c$, mm | 100 | 100 |
| $a$, mm | 60 | 64 |

Table 5. Results from the diagnosing session

# 7 CONCLUSION

The results from the performed tests and analysis shows that changes in eigenfrequencies and relative mode shapes forms an excellent base for the perfomance of damage detection in civil engineering structures.

The location of the crack is estimated exact and the crack length is overestimated by $\approx 6$ %. However the ladder mentioned deviation migth be estimated to large, because the measurement of the crack length was primitively measured by means of a Vernier gauge and a magnifying glass. This method will allways lead to a lower value for the exact crack length.

Furhter, better results could probably have been obtained if the calibration of the model for $\Delta\lambda$, which was recommended in section 3, had been perfomed.

Further the results shows, that the first eigenfrequency decrease as expected (see figure 1 and 6a) when the crack grow.

The expected increase in damping due to crack growth is not demonstrated in the same convincing manner. However the graph in figure 6b shows a growing tendency for $2a > 20mm$, where the cantilever contains a real fatigue crack. The drop at the start can be due to that the cantilever has been demounted from its fixture between the two measurements.

The ratio between the modal coordinates does not varies significant with the crack length (see figure 6c), which is due to that both measurement points are above the crack and the relatively small crack size.

# ACKNOWLEDGEMENTS

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